END

FILMED

DTIC

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS SYSTEMS RESEARCH LABORATORY

DEFENCE RESEARCH CENTRE SALISBURY
SOUTH AUSTRALIA

TECHNICAL REPORT

WSRL-0355-TR

AN ANALYSIS OF THE EFFECT OF MASS UNBALANCES AND
ASSEMBLY TOLERANCES ON THE PERFORMANCE OF THE
FOUR-AXIS STABILISED DIRECTOR MOUNT

J. HAYWARD

COPY No.

MAY 1984

AR-003-772

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS SYSTEMS RESEARCH LABORATORY

TECHNICAL REPORT

WSRL-0355-TR

AN ANALYSIS OF THE EFFECT OF MASS UNBALANCES AND ASSEMBLY TOLERANCES
ON THE PERFORMANCE OF THE FOUR-AXIS STABILISED DIRECTOR MOUNT

J. Hayward

S U M M A R Y

The four-axis, elevation over azimuth over pitch over
roll, director mount is analysed to determine the torques
generated by mass unbalances about the motor driven axes.
The effect of these on the motion of the stabilised
platform, and the effect of this motion, as well as that
of mechanical tolerances, on the accuracy of direction is
examined. The analysis is restricted to the case of the
fully erect system, firmly connected to a stationary,
level surface, but the method of approach could be
extended to more complex situations. The results provide
the basis for the design of small, lightweight, high
performance directors.

POSTAL ADDRESS: Director, Weapons Systems Research Laboratory,
Box 215', GPO, Adelaide, South Australia, 5001.

TABLE OF CONTENTS

Page

LIST OF APPENDICES

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

A necessary component of many weapon systems is the stabilised mechanical mount, used for directing the boresight of sensors and designators (optical, infrared, radar, etc) and the launching rails of weapons. (guided missiles, guns, etc). It must be able to point the boresight accurately in any direction (within a predetermined sector of three dimensional space) despite the inevitable changes in attitude adopted by the carrying vehicle. Two axes of motion between vehicle and boresight is the minimum number required, but practical considerations (such as the difficulty of providing unlimited motion about the axes, and the desire to reduce the inertia of rapidly slewing structures) often result in the use of three or four axes. Perhaps the most commonly used arrangement is that of the elevation-over-azimuth-over-pitch-over-roll, four-axis mount. This report considers only that type.

In many applications (particularly for sensors and designators) it is of great importance that the mount be small in size and light in weight, but nevertheless exhibit high pointing accuracy and rapid slewing of the boresight. These normally conflicting requirements can only be achieved if the design uses accurately fitted, light but strong, components, assembled into well-balanced structures, driven by servo motors of small size and weight and capable of only modest torque. If the requirements for fitting accuracy and static and dynamic balances are tighter than those achievable under normal manufacturing methods, it may be necessary to provide for adjustment-on-test of these parameters. In any case it is essential that the necessary conditions for balance and accuracy are clearly understood. This report is concerned with an analysis of these conditions.

Emphasis is given in this report to the particular configuration of two nominally identical, separately elevatable, sensors mounted symmetrically on a single structure, which rotates at a constant, significant angular rate about the (vertical) azimuth axis. Analysis is restricted to the case of operation on a stationary level vehicle, with small deviations from the vertical of the azimuth axis. This restriction results in mainly linear equations of motion which are directly solvable, but still gives a good in-sight into the problems of balance and accuracy. The approach could be extended, however, to cover other cases, such as the erection process, or operation on a moving vehicle.

The analysis begins in Section 2, where the ideal four-axis mount is described, followed by the definition of a sequence of five Cartesian coordinate systems attached one to each of the five separate solid structures interconnected by the four axes of roll, pitch, azimuth and elevation. Matrix equations are derived relating the coordinates of a single point referred to each of the five coordinate systems.

In Section 3 we derive general relations for the torque components, referred to one coordinate system, caused by the reaction force of a point mass, fixed in a second coordinate system (neither system being inertial), and subjected to gravitational and kinematic accelerations.

In Section 4 we define ten mass distribution parameters (moments) for each of the elevating, rotating and combined elevating and rotating assemblies. We calculate torque components about the elevating and rotating coordinate systems due to each of these parameters under varying angles of roll, pitch, azimuth and elevation. These torques are transformed to the motor driven axes of roll, pitch, azimuth and elevation, and the results are presented in tabular form.

Section 5 is concerned with the conditions of mass distribution for static and dynamic balance of the structure (no torques about the motor driven axes). The azimuth rate is assumed constant, and the elevation rate, roll angle and

pitch angle are assumed to be zero.   Ten separate parameters must be zeroed.

Sections 6, 7 and 8 are concerned with the behaviour of the roll and pitch error correction loops, under the assumption of constant azimuth rate, zero elevation rate and small pitch and roll errors.   The servo motor torque demands are assumed to be proportional to error and error rate, with a small unbalance permitted between the two loops.

It is shown in Section 6 that a uniform spiral motion of the azimuth axis about the vertical position generates a reaction torque (via the servo motors and the inertia parameters) which consists of three spiral components.   The main component is at the same angular rate, and in the same direction, as the motion.   One minor component (at much smaller amplitude) is at the same angular rate, but in opposite direction, and is proportional to the unbalance between the servo loop gains.   The second minor component (also at a much smaller amplitude) has an angular rate and direction depending on those of both the spiral motion and of the azimuth scanning rotation, and is proportional to the magnitude of one unbalanced mass distribution parameter. The magnitude of all three components depends on the angular rate of the spiral motion; and that of the main and the second minor component depends on the relative direction of the spiral and scanning motions.

The transient response of the roll and pitch system is motion in the absence of any externally applied forcing torque; and hence motion producing no reaction torque.   This is calculated in Section 7 by an iterative process. The spiral motion giving no main component of reaction torque is determined. This solution is then perturbed to correct it for the minor torque components. An infinite series of spiral motions is produced by this method, but, providing the unbalanced mass and servo gains are small, the amplitudes converge rapidly towards zero.

The residual (non-reaction) torques in the roll and pitch system result from mass unbalances interacting with the angular scanning motion about the azimuth axis, and combine to form a single spiral torque at the same rate and direction.   Its effect on the system is considered in Section 8.   Again an infinite, but rapidly converging series of spiral motions are produced, as forcing torques are matched with reaction torques.   The magnification of the forcing torque by the servo loop resonance is considered in terms of the torque demand from the roll and pitch axis motors.

Section 9 considers torques in the azimuth and elevation drives under the same steady conditions postulated for the preceding sections.

Section 10 considers the torques about all four motor-driven axes, due to a varying elevation angle combined with otherwise steady scanning conditions. Section 11 considers the same torques during azimuth axis acceleration.

If the azimuth axis is not truly vertical, due to roll and pitch correction errors, the true azimuth and elevation angles of the boresight differ slightly from the indicated values.   The differences are evaluated in Section 12.

Sections 13, 14, 15 and 16 consider the boresight errors and changes in motor torque demands resulting from deviations from the ideal geometrical arrangements of the four motor driven axes.

The results of the analysis, presented in tabular and graphical form, provide a firm basis for the design of the servo drives and of the procedure and facilities for achieving the necessary degree of mass balance.

2. IDEAL ARRANGEMENT OF MOUNT AXES

In this Section we describe and define the ideal arrangement of the axes and assemblies of the elevation-over-azimuth-over-pitch-over-roll, stabilised, four axis aerial mount. Deviations from the ideal arrangement are considered in later sections. A simple extension covers the case of two aerials, back to back, on two separate elevatable assemblies.

The basic concept of the ideal arrangement is that the motor-driven roll-correction, pitch-correction and azimuth axes always intersect at a common point, O, irrespective of the roll and pitch correction angles, η and ψ. Furthermore, the pitch and roll axes are always perpendicular, as are the pitch and azimuth axes.

The roll correction axis is adjusted to be horizontal, and parallel to the rear to front centre line of the carrying vehicle, when the latter is in its nominal level condition. The pitch correction axis is defined to be horizontal under the same vehicular condition and when the roll correction angle, η, is zero. The azimuth axis is defined to be vertical (pointing towards the zenith) when the vehicle is level, η is zero, and the pitch correction angle, ψ, is zero.

The Cartesian coordinate system with the b-suffix (figure 1(a)) is fixed to the base of the aerial mounting system. Its origin is the point, O, and its x-axis coincides with the roll correction axis. The y and z-axes coincide with the pitch and azimuth axes respectively when η and ψ are zero.

The Cartesian system with the g-suffix is fixed to the gimbal ring. Its origin is also O, its x-axis also coincides with the roll axis, and its y-axis coincides with the pitch axis.

The Cartesian system with the s-suffix is fixed to the stabilised platform (figure 1(b)). Its origin is also O, its y-axis also coincides with the pitch axis, and its z-axis coincides with the azimuth axis.

The Cartesian system with the r-suffix is fixed to the rotating assembly. Its origin is also O, its z-axis also coincides with the azimuth axis (figure 1(c)), and its y-axis is defined to be parallel to the motor driven elevation axis. The azimuth angle, θ, is defined to be zero when the x and y-axes coincide with those of the s-system.

The elevation axis is parallel to OY_r (figure 1(d)) and cuts the $X_r O Z_r$ plane at the point O_e, $(X_1, 0, Z_1)$ in r-system coordinates. The Cartesian system with the e-suffix is fixed to the elevating assembly. Its y-axis coincides with the elevation axis. The x-axis, $O_e X_e$, is defined to be the boresight direction of the associated aerial. The elevation angle, ϕ, is defined to be zero when the bore sight, and x-axis, are parallel to the x-axis in the r-system.

The Cartesian system fixed to the second (back to back) elevating assembly would coincide with that of the first elevating assembly if the latter were to be rotated through π radians about the OZ_r axis. O_e would occupy the point $(-X_1, 0, Z_1)$ in the r-system, and the x and y-axes would be oppositely directed to those in the r-system. An increase in ϕ tilts the boresight of the second assembly upwards.

We define the following rotation matrices and position column vector.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\eta & -\sin\eta \\ 0 & \sin\eta & \cos\eta \end{bmatrix} \quad B = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \qquad (1)$$

$$E = [X_1 \quad 0 \quad Z_1]^T$$

Then, by inspection of figure 1, the following relations apply to the coordinates of a single point.

$$P_b = A.P_g \qquad\qquad P_g = B.P_s$$

$$P_s = C.P_r \qquad\qquad P_r = E + D.P_e \qquad (2)$$

3. CALCULATION AND TRANSFORMATION OF TORQUE

3.1 General Cartesian axes

Consider a general set of Cartesian axes, OXYZ, moving in inertial space, as shown in figure 1(e). Consider also a point-mass, of magnitude, m, located at the time-dependent coordinates, x, y, z, with respect to these axes. The mass exerts a reaction force as a result of its absolute acceleration and of its presence in the earth's gravitational field. Suppose it is resolved into the components, F_x, F_y, F_z, parallel to the x, y and z-axes. We wish to determine the torque components, T_x, T_y, T_z, about the same axes.

An examination of the situation, using figure 1(e), shows that the following matrix relationship applies.

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} . \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

We define the column vectors of position, P, force, F, and torque, T, as well as the matrix, N(P), where:

$$P = [x \ y \ z]^T$$

$$F = [F_x \ F_y \ F_z]^T$$

$$T = [T_x \ T_y \ T_z]^T \qquad\qquad\qquad (3)$$

$$N(P) = \begin{bmatrix} 0 & [0 \ 0 \ -1].P & [0 \ 1 \ 0].P \\ [0 \ 0 \ 1].P & 0 & [-1 \ 0 \ 0].P \\ [0 \ -1 \ 0].P & [1 \ 0 \ 0].P & 0 \end{bmatrix}$$

Then the above relationship becomes:

$$T = N(P).F \qquad\qquad\qquad (4)$$

3.2 General inertial axes

Consider a general set of Cartesian axes, O'X'Y'Z', fixed in inertial space with O'Z' pointing towards the zenith. Suppose the same point-mass is located at the time-dependent coordinates, x', y', z', with respect to these axes. Suppose the same reaction force is resolved into the components, F_x', F_y', F_z', parallel to the corresponding inertial axes.

We define the column vectors, P' and F', and L, and the coordinate rotation matrix, R, such that:

$$P' = [x' \ y' \ z']^T$$

$$F' = [F_x' \ F_y' \ F_z']^T \qquad\qquad\qquad (5)$$

$$P' = L + R.P$$

Here, L defines the time-dependent offset between the origins of the two Cartesian systems, and R defines the time-dependent angles between the corresponding pairs of axes.

If g is the gravitational acceleration, then:

$$F' = - m.(G + \ddot{P}')$$

$$G = [0 \ 0 \ g]^T \qquad\qquad\qquad (6)$$

where the dots denote differentiation with respect to time.

The force-components transform isomorphically with the position coordinates under rotation of axes, and are unchanged under translation of axes. Therefore:

$$F' = R.F \tag{7}$$

Combining the relations (4),(6) and (7), we get the general relationship:

$$T = -m.N(P).R^{-1}.(G + \ddot{P}') \tag{8}$$

3.3 Properties of N(P)

It can be shown (by evaluation) that the inverse of the matrices, A, B and C (from equation (1)) are obtained by changing the sign of the angle parameter.

It can also be shown (by evaluation) that, if Q is a matrix of the type A, B or C (ie a coordinate rotation matrix), then for all F:

$$\left.\begin{aligned} N(Q.P) &= Q.N(P).Q^{-1} \\[2mm] N(P).P &= 0 \end{aligned}\right\} \tag{9}$$

3.4 Expansion of torque relation

Using equations (5) and (9):

$$N(P).R^{-1}.(\ddot{P}' - \ddot{L}) = R^{-1}.N(R.P).\frac{d}{dt}\{\frac{d}{dt}(R.P)\}$$

$$= R^{-1}.\frac{d}{dt}\{N(R.P).\frac{d}{dt}(R.P)\}$$

$$= (R^{-1}.\dot{R} + \frac{d}{dt})\{N(P).(R^{-1}.\dot{R}.P + \dot{P})\}$$

Therefore from equation (8):

$$T = -m.(R^{-1}.\dot{R} + \frac{d}{dt})\{N(P).(R^{-1}.\dot{R}.P + \dot{P})\} - m.N(P).R^{-1}.(G + \ddot{L}) \tag{10}$$

3.5 Axis transformation

We wish to determine the torque T'', given T and the axis transformation relationship:

$$P'' = K + Q.P \tag{11}$$

Then:

parameters, d and c_0.

8.3 Magnification of motor torque

The ratio of the magnitude of the servo motor torque, to that of the forcing torque (which equals the total deflection induced reaction torque) is of interest because significant magnification can occur if the forcing angular rate approaches the servo resonance frequency. We consider the simple case of negligible servo unbalance (u and f are zero) and one spiral component of forcing torque.

The expression for motor torque is given in equation (27). If values for U and F are substituted from equation (29), and for $(\eta + j.\psi)$ from equations (31) and (42), we find that the ratio of motor torque to forcing torque is given by:

$$T_M/T_f = -e^{st}.\{\omega_0{}^2 + g.V/(J+Q) + 2c_0.\omega_0.s\}/G(s) \tag{48}$$

The magnitude of this expression is plotted in figure 4, parts (a) to (d), for $(s = j.\dot\theta)$, as a function of $\omega_0/\dot\theta$, and for several values of the parameters, d and c_0. V is assumed to be zero. For (d = 0.5) the ratio is identically unity.

Note that:

(a) The parameter, V, appears in equation (48) only because of the definition of ω_0 in equation (29).

(b) For values of d greater than 0.5, the system is in stable equilibrium even without the motor torque, and the magnification is less than unity.

(c) For values of d less than 0.5, the system is unstable without the motor torque, and magnifications of up to 3 dB can occur.

(d) If the servo is to be in control of the system, and particularly if d is less than 0.5, the value of ω_0 must be at least three times the value of $\dot\theta$.

9. STEADY STATE TORQUES IN AZIMUTH AND ELEVATION

9.1 General

In this Section we consider torques about the azimuth and elevation axes under the same steady-state conditions previously assumed, viz $(\ddot\theta = \ddot\phi = \dot\phi = 0, \eta = \psi = 0)$. The assumptions imply that these torques are resisted by the respective servo motors with no change in motion (ie infinite stiffness).

9.2 Azimuth torque (T_{rz})

We use the combined rotating and elevating assembly parameters (Subsection 4.5) derived from Table 4, to find that:

$$s = j.\dot{\theta}$$

$$T_f = j.L.\dot{\theta}^2.e^{j\gamma}$$
(43)

(b) For the H-term from equation (40):

$$s = j.\dot{\theta}$$

$$T_f = j.H.g.e^{j\alpha}$$
(44)

(c) For the K-term from equation (35), excited by the spiral deflection defined in equation (31):

$$s' = s^* + 2j.\dot{\theta}$$

$$T_f' = -K.A^*.e^{-j\beta}.\{(s^*)^2 + 2j.\dot{\theta}.s^*\}$$
(45)

(d) For the K-term from equation (35), excited by the main spiral deflection produced by the forcing torque T_f:

$$s' = s^* + 2j.\dot{\theta}$$

$$(T_f')^* \simeq -K.T_f.e^{j\beta}.(s^2-2j.\dot{\theta}.s)/\{(J+Q).G(s)\}$$
(46)

$$(A')^* \simeq K.T_f.e^{j\beta}.(2j.\dot{\theta}.s-s^2)/\{(J+Q)^2.G(s).G^*(s^*+2j.\dot{\theta})\}$$

Note that, if $s = j.\dot{\theta}$, then:

$$s' = j.\dot{\theta}$$

$$(2j.\dot{\theta}.s-s^2) = -\dot{\theta}^2$$
(47)

$$(s^*+2j.\dot{\theta}) = j.\dot{\theta}$$

and the output angular frequency and direction are the same as the input. This does not apply for $(s = -j.\dot{\theta})$, for which the output occurs at $(s'= 3j.\dot{\theta})$.

Figure 3, parts (a) to (e), show the value of the magnitude of $G(s)$ (normalised to $\dot{\theta}^2$) plotted as a function of ω_0 (normalised to $\dot{\theta}$) for $(s = j.\dot{\theta})$, $(s = -j.\dot{\theta})$ and $(s = 3j.\dot{\theta})$, and for several values of the

7.3 Perturbations

Newton's approximation, or some other iterative method, can be used to correct the value of s, determined in Sub-section 7.2, to make the function (33) zero for finite values of u and f. This converges rapidly for values of the order of 0.2 or less.

If the value of K is small but finite, a spiral torque can be determined from equation (35) for each of the four spiral deflection terms (two values of s, and their conjugates). These are then regarded as forming torques, which produce spiral deflection pairs at the same angular frequency, as discussed in Section 8. These in turn produce further reactions from the linear reaction section. Normally the infinite sequence of terms converges rapidly to negligible amplitude. (Note that the terms from equation (35) need a sign reversal, to convert them to forcing torques).

8. STABILISATION SYSTEM FORCED RESPONSE

8.1 General

An examination of Table 6 shows that, under the same conditions assumed in Sections 6 and 7 (ie $\ddot{\theta} = \ddot{\phi} = \dot{\phi} = 0$, and $\eta \simeq \psi \simeq 0$), there are still some torques not accounted for as deflection induced torques (Section 6). These forcing torques are independent of any (small) spiral deflection motions, and are given by:

$$(T_{roll} + j.T_{pitch}) = j.L.\dot{\theta}^2.e^{j\theta}1 + j.H.g.e^{j\theta}h \qquad (40)$$

These can be generalised to the form:

$$(T_{roll} + j.T_{pitch}) = T_f.e^{st} \qquad (41)$$

where, in this case, $(s = j.\dot{\theta})$ and T_f is the appropriate complex quantity for equivalence of equations (40) and (41).

The forcing torque must always be balanced by a motion induced reaction torque (from equation (33)). Thus s is defined by the forcing torque, and:

$$A = \frac{T_f}{(J+Q).\{G(s) - f(u,f)\}} \qquad (42)$$

where f(u,f) represents the right-most term in equation (33).

The induced spiral deflection terms are then defined by equation (32). The main term is clearly at the same angular rate, and in the same direction as the forcing torque.

8.2 Special cases

(a) For the L-term from equation (40):

This is applicable independently to the roll and pitch loops, because there is no cross coupling. Critical damping occurs with ($c_0 = 1$).

(b) If $\dot{\theta} \neq 0$, we substitute ($s = \alpha + j.\omega$) in (36) and equate the real and imaginary parts separately to zero. Thus:

$$(\omega - d.\dot{\theta}).(\alpha + c_0.\omega_0) + c_0.\omega_0.d.\dot{\theta} = 0$$

$$-(\omega - d.\dot{\theta})^2 + (\alpha + c_0.\omega_0)^2 + d^2.\dot{\theta}^2 + \omega_0^2.(1 - c_0^2) = 0$$

Note that ($\omega = d.\dot{\theta}$) is not a solution, else ($c_0.\omega_0 = 0$) and α is imaginary. Therefore:

$$\alpha = -c_0.\omega_0.\omega/(\omega - d.\dot{\theta}) \qquad (38)$$

and, multiplying by $-(\omega - d.\dot{\theta})^2$ and rearranging:

$$2(\omega - d.\dot{\theta})^2 = d^2.\dot{\theta}^2 + \omega_0^2.(1 - c_0^2)$$

$$+ \{(d^2.\dot{\theta}^2 + \omega_0^2.(1 - c_0^2))^2 + 4c_0^2.\omega_0^2.d^2.\dot{\theta}^2\}^{\frac{1}{2}}$$

$$= 2d^2.\dot{\theta}^2 + \omega_0^2.(1 - c_0^2) - d^2.\dot{\theta}^2$$

$$+ \{(\omega_0^2.(1 - c_0^2) - d^2.\dot{\theta}^2)^2 + 4\omega_0^2.d^2.\dot{\theta}^2\}^{\frac{1}{2}} \qquad (39)$$

Note that, from equations (39),(38):

(1) $(\omega - d.\dot{\theta})^2 > d^2.\dot{\theta}^2$

  Thus: $\omega > 2d.\dot{\theta}$ or $\omega < 0$

(2) In both cases, $\alpha < 0$, and transients decay to zero with increase in time.

(3) $\left| \dfrac{\omega}{\alpha} \right| = \left| \dfrac{\omega - d.\dot{\theta}}{c_0.\omega_0} \right|$ takes the same value for both solutions, so the spirals are symmetrical, although one is executed much faster than the other.

Figure 2 shows the magnitude of ($\omega - d.\dot{\theta}$) from equation (39), plotted in a normalised form, as a function of $\omega_0/d.\dot{\theta}$ and c_0. It can be seen that, for large values of c_0, there is a forward spiral response at an angular rate of just greater than $2 d.\dot{\theta}$, together with a reverse spiral response at an angular rate close to zero. The directions are unchanged but the rates increase considerably as the damping factor, c_0, is reduced, depending also on the stiffness of the servo loops (ω_0).

Then equation (30) becomes:

$$-(T_{roll}+j.T_{pitch})/(J+Q)$$

$$\simeq A.e^{st}.\{G(s) - (u.\omega_0^2 + 2f.c_0.\omega_0.s)^2/(G(s) + 4j.d.\theta.s)\} \tag{33}$$

6.4 Non-linear reaction to spiral deflection

We now consider the contribution of the K-term to the spiral torque. From Table 6:

$$-(T_{roll}+j.T_{pitch}) = K.e^{2j.\theta}k.\{(\ddot{\eta}-j.\ddot{\psi}) + 2j.\dot{\theta}.(\dot{\eta}-j.\dot{\psi})\} \tag{34}$$

$$= K.A^*.e^{(s^*t+2j.\theta_k)}.\{(s^*)^2 + 2j.\dot{\theta}.s^*\} \tag{35}$$

under the single spiral deflection (equation (31)).

The significance of this component will be considered later.

7. STABILISATION SYSTEM TRANSIENT RESPONSE

7.1 General

The transient response of any system is obtained by equating to zero, the total deflection-induced torque. In this case it is the sum of the functions (30) and (34). A closed solution can not be obtained directly, However we can obtain a solution by assuming that K, u and f are zero, and then perturbing it to take into account the small finite values of these parameters.

7.2 Unperturbed solution

We assume the form of the solution is as defined in equation (32). Then, with (K = u = f = 0), from equations (33) and (35), the torque is zero if:

$$G(s) = 0$$

This can be rearranged algebraically to give:

$$(s-j.d.\theta+c_0.\omega_0)^2 + 2j.c_0.\omega_0.d.\theta + d^2.\theta^2 + \omega_0^2.(1-c_0^2) = 0 \tag{36}$$

(a) If $\dot{\theta} = 0$, we get the well known result:

$$s = -c_0.\omega_0 \pm \omega_0.(c_0^2-1)^{\frac{1}{2}} \tag{37}$$

$$\omega_0{}^2 = \frac{U - g.V}{J + Q}$$

$$c_0 = \frac{F}{2\{(U-g.V).(J+Q)\}^{\frac{1}{2}}}$$

$$d = \frac{J}{J+Q}$$

(29)

and assuming that $\frac{g.V}{U} \simeq 0$, equation (28) becomes:

$$-(T_{roll}+j.T_{pitch})/(J+Q) \simeq (\ddot{\eta}+j.\ddot{\psi}) + 2(c_0.\omega_0-j.d.\dot{\theta}).(\dot{\eta}+j.\dot{\psi})$$

$$+ \omega_0{}^2.(\eta+j.\psi)$$

$$+ u.\omega_0{}^2.(\eta-j.\psi) + 2f.c_0.\omega_0.(\dot{\eta}-j.\dot{\psi}) \qquad (30)$$

6.3 Linear reaction to spiral deflection

Suppose: $\eta+j.\psi = A.e^{st}$

$$s = \alpha+j.\omega$$

(31)

From parts (a) and (b) of figure 1, this is an anti-clockwise spiral motion of the point Z_r (looking down towards the origin) about the point, Z_b, with angular frequency, ω, and logarithmic amplitude increment, α. A should be considered as complex, to represent the amplitude and phase angle at $t = 0$. Replacing s with its complex conjugate reverses the direction of the spiral.

If we substitute equation (31) into equation (30) we obtain a major term in e^{st} (being a spiral torque in the same direction as the spiral deflection, and with the same angular frequency and increment) and a minor term in e^{s^*t} (being a spiral torque in the reverse direction with the same angular frequency and increment) due to the unbalanced servo motor gains u and f. It is more useful to add a small spiral deflection in the reverse direction to cancel the reverse spiral torque. This is achieved if:

$$(\eta+j.\psi) = A.e^{st} - B^*.e^{s^*t}$$

$$G(s) = s^2 + 2(c_0.\omega_0-j.d.\dot{\theta}).s + \omega_0{}^2$$

$$B = A.(u.\omega_0{}^2 + 2f.c_0.\omega_0.s)/(G(s) + 4j.d.\dot{\theta}.s)$$

(32)

$$2K_e = \sum_e m.(z_e')^2 - \sum_e m.(x_e')^2 \tag{26}$$

Thus K_e is proportional to the difference between the principal moments of inertia in the $X_e Z_e$- plane. For $K_e = 0$, the two principal moments of inertia must be equal (and the e'-system axes are principal for all values of ϕ).

6. DEFLECTION INDUCED TORQUES IN THE STABILISATION SYSTEM

6.1 General

In this Section we examine the torques present in the roll and pitch correction system due to the deflection of the azimuth axis from the vertical position $(\eta + j.\psi)$ and its time derivatives, under the assumptions $(\theta = \phi = \dot{\phi} = 0)$ and $(\eta \simeq \psi \simeq 0)$. These consist of gravitational and inertial reactions (already determined, and listed in Table 6) and servo motor supplied torques.

We assume that the roll and pitch correction servo motors produce torques, proportional to the deflection error and its first time derivative, and in the direction to reduce that error. We wish also to examine the effect of small differences in the constants of proportion, between the roll and pitch systems. We therefore postulate the following relations for motor torque:

$$\left. \begin{aligned}
T_{roll} &= -U.(1+u).\eta - F.(1+f).\dot{\eta} \\
T_{pitch} &= -U.(1-u).\psi - F.(1-f).\dot{\psi} \\
-(T_{roll} + j.T_{pitch}) &= U.(\eta+j.\psi) + F.(\dot{\eta}+j.\dot{\psi}) \\
&\quad + U.u.(\eta-j.\psi) + F.f.(\dot{\eta}-j.\dot{\psi})
\end{aligned} \right\} \tag{27}$$

6.2 Linear reaction torque

From Table 6, and the assumption $(\dot{\theta} = 0)$ all reaction terms are linear except those in K. For this sub-section we assume that $K = 0$. Then the total reaction torque is given by:

$$-(T_{roll}+j.T_{pitch}) = (J+Q).(\ddot{\eta}+j.\ddot{\psi}) + (F-2j.J.\dot{\theta}).(\dot{\eta}+j.\dot{\psi})$$
$$+ (U-g.V).(\eta+j.\psi)$$
$$+ u.U.(\eta-j.\psi) + f.F.(\dot{\eta}-j.\dot{\psi}) \tag{28}$$

Making the normalising substitutions:

(2) The conditions involving H_e and H_r are the obvious ones that the centre of mass of the elevating assemblies be on the elevation axis, and that of the rotating plus both elevating assemblies be on the azimuth axis. The structure is then statically balanced. These conditions can be easily met in practice, with proper mechanical design and with adjustable weights for fine trimming.

(3) The conditions involving L_e and L_r are those for dynamic balance of the elevating, and total rotating assemblies, about the elevation and azimuth axes, respectively. These conditions are less easily met, and require careful attention.

(4) The condition involving K_e is very difficult to achieve in practice. Its physical implication is discussed in detail in Sub-section 5.6. With two elevating assemblies, the effect about the roll and pitch axes tends to cancel, if the two values of ϕ are not too dissimilar. This may not be a reasonable constraint, however.

(5) If torque balance is required for small constant values of ψ and η (as well as at the zero values) then the Tables 6, 1, 3, 4 and 2 show the extra requirement to be:

$$V_r + M_e.Z_1 = 0 \tag{25}$$

The e-suffix term implies the difference between the two elevating assemblies.

5.6 Physical significance of K_e

The Cartesian system with the e'-suffix is defined by:

$$P_e' = D.P_e$$

Then, by evaluation:

$$\sum_e m.(x_e')^2 = J_e - K_e.\cos(2\phi - \beta_e)$$

$$\sum_e m.(z_e')^2 = J_e + K_e.\cos(2\phi - \beta_e)$$

$$\sum_e m.x_e'.z_e' = -K_e.\sin(2\phi - \beta_e)$$

We choose $2\phi = \beta_e$, making the cross product function, zero. Hence the e'-system becomes a principal set of axes, and the other two functions take the maximum and minimum possible values. Then:

5.2 Roll and pitch torques

$$(T_{roll} + j.T_{pitch}) = j.\dot{\theta}^2.e^{j\theta}.\{(L_r.\cos\gamma_r - K_e.\sin2\phi_k$$

$$+ M_e.X_1.Z_1 + X_1.H_e.\sin\phi_h + Z_1.H_e.\cos\phi_h)$$

$$+ j.(L_r.\sin\gamma_r + L_e.\sin\phi_l + Z_1.V_e)\}$$

$$+ j.g.e^{j\theta}.\{(H_r.\cos\alpha_r + H_e.\cos\phi_h + M_e.X_1)$$

$$+ j.(H_r.\sin\alpha_r + V_e)\} \tag{21}$$

For this expression to be identically zero for arbitrary values of ϕ, the requirements are:

$$
\left.
\begin{array}{l}
H_e = K_e = L_e = 0 \\[4pt]
H_r.\cos\alpha_r + M_e.X_1 = 0 \\[4pt]
H_r.\sin\alpha_r + V_e = 0 \\[4pt]
L_r.\cos\gamma_r + M_e.X_1.Z_1 = 0 \\[4pt]
L_r.\sin\gamma_r + Z_1.V_e = 0
\end{array}
\right\} \tag{22}
$$

5.3 Azimuth torque (T_{rz})

This torque is automatically zero, so no special requirements apply.

5.4 Elevation torque (T_{ey})

$$T_{elev} = \dot{\theta}^2.\{X_1.H_e.\sin\phi_h - K_e.\sin2\phi_k\} + g.H_e.\cos\phi_h \tag{23}$$

For this expression to be identically zero for arbitrary values of ϕ, the requirements are:

$$H_e = K_e = 0 \tag{24}$$

5.5 Discussion of results

(1) The conditions for e-suffix terms in equation (22) imply the difference between the two elevating assemblies, because $e^{j\theta}$ terms tend to cancel (Sub-section 4.6). But if different values of ϕ are allowed for the two units, cancellation is not possible and the zero condition for H_e, K_e and L_e apply to each unit individually. The conditions in equation (24) apply individually in either case.

$$T_{roll} \quad = \quad T_{gx} \quad = \quad [1 \ 0 \ 0 \ ].B.C.T_{r}$$

$$T_{pitch} \quad = \quad T_{sy} \quad = \quad [0 \ 1 \ 0 \ ].C.T_{r}$$

Both T_{roll} and T_{pitch} are real variables because B,C and T_{r} are real. (Note that complex terms in T_{r} occur as the sum of conjugate pairs). Therefore:

$$
\left.
\begin{aligned}
T_{roll} \quad &= \quad Re\{T_{roll} + j.T_{pitch}\} \\[2mm]
T_{pitch} \quad &= \quad Im\{T_{roll} + j.T_{pitch}\} \\[2mm]
(T_{roll} + j.T_{pitch}) \quad &= \quad [e^{j\theta} \ j.e^{j\theta} \ \psi \ ].T_{r}
\end{aligned}
\right\} \qquad (20)
$$

Table 5 lists the contribution of the elevating assembly to $(T_{roll} + j.T_{pitch})$, evaluated from Table 3, using equation (20). Table 6 lists the contribution of the combined assemblies, at fixed elevation, and is derived from Table 4 by deleting the r-suffix from the mass distribution parameters (as discussed in Sub-section 4.5).

5.   CONDITIONS FOR BALANCE WITH STEADY OPERATION

5.1 Steady operation

For a continuously scanning radar, it is normally very desirable that all torques about the four motor driven axes (roll, pitch, azimuth and elevation) should be zero when:

(a)   the aerial mount is fixed to a level surface

(b)   the rotating assembly is fully erect (ie $\psi = \eta = 0$)

(c)   there is a fixed, but arbitrary elevation angle (ϕ)

(d)   there is a constant (or zero) scan rate (θ).

Under these conditions, the torques, obtained from Tables 6, 1, 3, 4 and 2 become as shown in the next sub-sections.

of torque in the θ-column requires that:

$$L.\cos\gamma = L_r.\cos\gamma_r + M_e.X_1.Z_1 - K_e.\sin2\phi_k + H_e.(X_1.\sin\phi_h + Z_1.\cos\phi_h)$$

All such relationships are listed in Table 1.

Note that most of the non-suffixed parameters are functions of ϕ.

4.6 Contribution of second elevating assembly

As discussed in Section 2, the second elevating assembly is (ideally) equivalent to the first assembly rotated by π radians about the z-axis of the r-system. Its contribution to its own T_e is therefore given by the terms in Table 2, except that each θ (including any in the column heading) is replaced by $(\theta+\pi)$. The effect of this is to change the sign of entries in those columns containing $\exp(\mp j.\theta)$ in the heading.

The contribution to T_r is determined indirectly. Clearly, the contribution to T_s is obtained by replacing θ by $(\theta+\pi)$ wherever it occurs. Thus the contribution to T_r is given by:

$$C^{-1}.(C'.T_r') = \begin{bmatrix} -T_{rx}' \\ -T_{ry}' \\ T_{rz}' \end{bmatrix} \tag{19}$$

where the prime denotes replacement of θ by $(\theta+\pi)$. Thus the rule for Table 3 becomes: change the sign of elements in the x and y components of torque; then change the sign of elements in the columns containing $\exp(\mp j.\theta)$ in the heading.

Note that this algorithm for T_r is equivalent to changing the sign for each occurrence (in element and row and column heading) of H_e, V_e, X_1, ϕ, ϕ_h, ϕ_1, ϕ_k. Executing this latter algorithm on the relationships for the non-suffixed mass distribution parameters in Table 1, results in a change of sign for contributions to those parameters which change sign when both x_r and y_r are changed in sign, ie $H.\cos\alpha$, $H.\sin\alpha$, $L.\cos\gamma$ and $L.\sin\gamma$. This is as would be expected, since the x and y directions in the r-system are reversed for the second elevating assembly.

Final sign reversals for contributions to T_r and T_s reflect the balancing effect of the second assembly upon the first, due to the symmetrical structure and location about the azimuth axis.

4.7 Torque about roll and pitch axes

From figure 1 and equation (12) it follows that:

From equations (2) and (11):

$$K = E$$
$$Q = D$$
$$\left. \right\} \quad (17)$$

Items (k) to (u) of Appendix I are additional key results in the evaluation of equation (16). The final results are shown in Table 3 (in the same format as used in Table 2), except that terms in M_e (ie those which consider the elevating assembly as a point mass at its origin) have been included in Table 4, together with the contributions of the rotating assembly.

Note that the z-component of T_r is the torque (due to the elevating assembly) to be resisted by the azimuth servo motor.

4.4 Contribution of rotating assembly to T_r

The torques generated about the axes of the r-system due to the reactions from the masses forming the rotating (but not elevating) assembly are given by equation (14), where in this case from equations (2) and (5):

$$P' = P_b = A.B.C.P_r$$
$$P = P_r$$
$$L = 0$$
$$R = A.B.C$$
$$\left. \right\} \quad (18)$$

Items (v) to (y) of Appendix I are additional key results in the evaluation, which also uses the set of ten mass distribution parameters for the rotating assembly (indicated by the r-suffix) listed in Table 1. The final results are shown in Table 4, which includes in addition, the terms in M_e from the elevating assembly, derived in Sub-section 4.3.

Note that columns involving ϕ and its derivatives have been deleted from the table. The mass distribution of the rotating assembly (alone) is obviously independent of the elevation angle of the elevating assembly.

Note also that the z-component of T_r is the torque (due to the rotating assembly alone) to be resisted by the azimuth servo motor.

4.5 Combined contribution to T_r

It is obvious that, at a fixed elevation angle (ϕ), the elevating assembly can be considered as an extension of the rotating assembly. Therefore we can define a combined set of mass distribution parameters, M, J, Q, K, L, H, V, α, β and γ, which behave isomorphically with the r-suffixed equivalents, but which include the contribution from the elevating assembly at a fixed, but arbitrary, elevation.

The relationship between the non-suffixed parameters and the r and e-suffixed ones can be determined from the data in Tables 3 and 4, by equating total torques in each axis-column. For example, the x-component

$$T_e = -(R^{-1} \cdot \dot{R} + \frac{d}{dt}) \left\{ \sum_c m.N(P).(R^{-1} \cdot \dot{R}.P + \dot{P}) \right\} - \sum_e m.N(P).R^{-1}.(G + \ddot{L}) \quad (14)$$

where from equations (2) and (5):

$$\left. \begin{array}{rcl} P' = P_b & = & A.B.C.(E + D.P_e) \\[2mm] P = P_e & & \\[2mm] L & = & A.B.C.E \\[2mm] R & = & A.B.C.D \end{array} \right\} \quad (15)$$

This can be evaluated directly, using the mass-distribution parameters defined in Table 1. Between them, the ten parameters associated with the elevating assembly (indicated by the e-suffix) cover the entire range of zero order, first order and second order moments of mass, measured in the coordinate system fixed to that assembly. The form of the parameters has been chosen to suit the evaluation of the torques.

Items (a) to (j) of Appendix I are key results in the step by step evaluation of equation (14). The final results are presented, in Table 2, in the form of a matrix. Each entry in the matrix is to be multiplied by its row and column headers, before being accumulated into the total component of the torque.

The first five columns contain dynamic entries due to the motions of rotation (θ) and elevation (ϕ). The ninth and tenth columns contain static entries due to gravity. Columns 6, 7 and 8 contain dynamic entries due to the changing values of roll (η) and pitch (ψ) correction. Note that each of columns 6, 7, 8 and 10, represent two separate sets of entries, denoted by the upper and lower sign (where a choice is provided). The upper sign is used for one set; the lower sign for the other. Wherever only one sign is given, it applies to both sets.

The complex variable form of column headings simplifies evaluation and presentation, and will be found to be useful in later Sections.

Note that the y-component of T_e (as listed in Table 2) is the torque to be resisted by the elevation servo motor.

4.3 Contribution of elevating assembly to T_r

Consider the torques generated by the reactions of the masses located on the elevating assembly, about the axes of the r-system. Evaluation is easier if we use the transformation relationship (12), rather than the direct relationship (10). Thus:

$$T_r = Q.T_e - \sum_e m.N(K).Q.R^{-1}.(G + \ddot{P}') \quad (16)$$

where the variables are defined in equations (15) and (17).

$$P' = L + R.Q^{-1}.(P'' - K)$$

and from equation (8):

$$T'' = -m.N(P'').(R.Q^{-1})^{-1}.(G + \ddot{P}')$$

$$= -m.N(K + Q.P).Q.R^{-1}.(G + \ddot{P}')$$

$$= Q.T - m.N(K).Q.R^{-1}.(G + \ddot{P}') \tag{12}$$

Note that the second term of equation (12) contains no second order terms in the elements of a position vector. All second order terms are transformed within the first term of equation (12).

4. TORQUES DUE TO MASS UNBALANCE

4.1 General

In this Section we consider the torques acting about various axes, with the aerial base levelled on firm ground, and with the azimuth axis approximately vertical. In this case, the roll and pitch correction angles, η and ψ, are small enough that second order terms in these quantities can be neglected. Then:

$$\left.\begin{array}{l} A.B \simeq I + X \\[2em] X = \begin{bmatrix} 0 & 0 & \psi \\ 0 & 0 & -\eta \\ -\psi & \eta & 0 \end{bmatrix} \\[2em] (A.B)^{-1} \simeq I - X \\[2em] \dfrac{d}{dt}(A.B) \simeq \dot{X} \end{array}\right\} \tag{13}$$

The torques considered are caused by mass unbalances within the structure of the rotating and elevating assemblies. To calculate them, we must sum the effect of reaction forces over the entire rotating and elevating assemblies.

On the assumption that the b-system of axes is inertial, with the z-axis pointing towards the zenith, this system is isomorphic with the singly primed inertial system discussed in Section 3, and the torques can be evaluated using the relationships derived therein.

4.2 Contribution of elevating assembly to T_e

Consider the torques generated about the axes of the e-system due to reactions from the masses forming the normal elevating assembly. From equation (10):

$$T_{az} = \tfrac{1}{2}L.\{e^{-j\theta}l.(\ddot{\eta}+j.\ddot{\psi}) + e^{j\theta}l.(\ddot{\eta}-j.\ddot{\psi})\}$$

$$- \tfrac{1}{2}g.H.\{e^{-j\theta}h.(\eta+j.\psi) + e^{j\theta}h.(\eta-j.\psi)\}$$

$$= Re\{L.e^{-j\theta}l.(\ddot{\eta}+j.\ddot{\psi}) - g.H.e^{-j\theta}h.(\eta+j.\psi)\} \tag{49}$$

where Re denotes "the real part of".

Considering a single spiral deflection component, as defined in equation (31):

$$T_{az} = |A|.Re\{L.s^2.e^{\{st-j(\theta_1-\angle A)\}} - g.H.e^{\{st-j(\theta_h-\angle A)\}}\} \tag{50}$$

In the common case $(s = j.\dot{\theta})$:

$$T_{az} = -|A|.L.\dot{\theta}^2.\cos(\angle A-\gamma) - |A|.g.H.\cos(\angle A-\alpha) \tag{51}$$

Both terms consist of a steady torque, of magnitude dependent on the phase relationship between the relevant mass-unbalance and the deflection component.

If $(s = -j.\dot{\theta})$ or $(s = 3j.\dot{\theta})$ the torque components are sinusoidal, and at an angular frequency of $2\dot{\theta}$.

9.3 Elevation torque (T_{ey})

From Table 2, and assuming the same single spiral deflection (equation (31)):

$$T_{elev} = \dot{\theta}^2.(X_1.H_e.\sin\phi_h-K_e.\sin2\phi_k) + g.H_e.\cos\phi_h$$

$$+ Re[A.e^{(st-j\dot{\theta})}.\{j.s^2.(2J_e+X_1.H_e.\cos\phi_h + Z_1.H_e.\sin\phi_h-j.L_e.\cos\phi_1)$$

$$+ 2\theta.s.(J_e+X_1.H_e.\cos\phi_h-K_e.\cos2\phi_k-j.L_e.\cos\phi_1)-j.g.H_e.\sin\phi_h\}] \tag{52}$$

For $(s = j.\dot{\theta})$:

$$T_{elev} = \dot{\theta}^2.(X_1.H_e.\sin\phi_h-K_e.\sin2\phi_k) + g.H_e.\cos\phi_h$$

$$+ \dot{\theta}^2.|A|.(Z_1.H_e.\sin\phi_h-X_1.H_e.\cos\phi_h+2K_e.\cos2\phi_k).\sin\angle A$$

$$+ \dot{\theta}^2.|A|.L_e.\cos\phi_1.\cos\angle A + |A|.g.H_e.\sin\phi_h.\sin\angle A \tag{53}$$

For $(s = j.\omega)$, the term in J_e becomes:

$$2|A|.\omega.(\omega-\dot\theta).\sin(\omega.t-\theta+\angle A).J_e \tag{54}$$

Note that, for the second elevating assembly, θ is incremented by π in equation (52), and all terms in A take a reversed sign.

10. TORQUES DUE TO VARYING ELEVATION ANGLE

Tables 5, 3 and 2 list all significant torques due to $\dot\phi$ and $\ddot\phi$, about the roll and pitch, azimuth and elevation axes, respectively. Major terms, and those of first order of smallness, are given below.

$$(T_{roll}+j.T_{pitch})$$

$$\simeq j.e^{j\theta}.\{\ddot\phi.(2J_e+X_1.H_e.\cos\phi_h+Z_1.H_e.\sin\phi_h+j.L_e.\cos\phi_1)$$

$$- \dot\phi^2.(X_1.H_e.\sin\phi_h-Z_1.H_e.\cos\phi_h+j.L_e.\sin\phi_1)$$

$$+ 2j.\dot\theta.\dot\phi.(J_e+Z_1.H_e.\sin\phi_h+K_e.\cos2\phi_k)\} \tag{55}$$

$$T_{az} \simeq - \ddot\phi.L_e.\sin\phi_1-\dot\phi^2.L_e.\cos\phi_1$$

$$+ 2\dot\theta.\dot\phi.(X_1.H_e.\sin\phi_h-K_e.\sin2\phi_k)$$

$$+ 2\dot\phi.|A|.J_e.Re[s.e^{(st-j\theta+j\angle A)}] \tag{56}$$

$$T_{elev} = 2J_e.\ddot\phi \tag{57}$$

Note that all terms in equation (55) tend to cancel when two elevating assemblies have the same variation of elevation angle. Only the term in $|A|$ does so, in equation (56).

11. TORQUES DUE TO AZIMUTH ACCELERATION

Tables 6, 4 and 2 list the significant torques due to $\ddot\theta$. Major terms, and those of first order of smallness, are given below.

$$(T_{roll}+j.T_{pitch})$$

$$\simeq L.\ddot\theta.e^{j\theta}1 - 2J.\ddot\theta.|A|.Im[e^{(st+j\angle A)}] \tag{58}$$

$$T_{az} \simeq -2J.\ddot{\theta} \tag{59}$$

$$T_{elev} \simeq \ddot{\theta}.L_e.\sin\phi_1 \tag{60}$$

12. EFFECT OF ROLL AND PITCH ERRORS ON BORESIGHT

12.1 Boresight error

As discussed in Section 2, the aerial boresight is assumed to be the axis, $O_e X_e$, which is normal to the elevation axis at point, O_e, and parallel to the XY-plane of the r-system when $\phi = 0$ (see figure 1(d)).

Suppose the boresight passes through a fixed target, located at the point, P_b, under two separate conditions. Firstly, with no roll and pitch errors:

$$A.B = I$$

$$C = C$$

$$D = D$$

$$P_e = r.\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = r.Z$$

Secondly, with finite roll and pitch errors:

$$A.B = A.B$$

$$C = C + \Delta C$$

$$D = D + \Delta D$$

$$P_e = (r + \Delta r).Z$$

Using equation (2) to determine P_b for each case, and then equating the results:

$$A.B.(C+\Delta C).(E + (D+\Delta D).(r+\Delta r).Z) = C.(E + D.r.Z)$$

Therefore:

$$D^{-1}.(I + C^{-1}.\Delta C).D.\{(I + D^{-1}.\Delta D).(r+\Delta r).Z + D^{-1}.E\}$$

$$= D^{-1}.C^{-1}.(A.B)^{-1}.C.D.(r.Z + D^{-1}.E) \tag{61}$$

Assuming that $\Delta r/r$ and the elements of $D^{-1}.E/r$ are negligibly small compared with unity, then equation (61) reduces to:

$$D^{-1}.C^{-1}.\Delta C.D.Z + D^{-1}.\Delta D.Z + D^{-1}.C^{-1}.\Delta C.D.D^{-1}.\Delta D.Z$$

$$\simeq D^{-1}.C^{-1}.\{(A.B)^{-1}-I\}.C.D.Z \tag{62}$$

Noting that:

$$\cos(\theta+\Delta\theta) = \cos\Delta\theta.\cos\theta + \sin\Delta\theta.\frac{d}{d\theta}(\cos\theta)$$

$$\sin(\theta+\Delta\theta) = \cos\Delta\theta.\sin\theta + \sin\Delta\theta.\frac{d}{d\theta}(\sin\theta)$$

$$C + \Delta C = \cos\Delta\theta.C + \sin\Delta\theta.\frac{dC}{d\theta}$$

$$C^{-1}.\Delta C = (\cos\Delta\theta-1).I + \sin\Delta\theta.C^{-1}.\frac{dC}{d\theta}$$

and using equation (1), the left hand side of equation (62) becomes:

$$\begin{bmatrix} \cos\Delta\theta.\cos\Delta\phi - 1 \\ \sin\Delta\theta.\cos(\phi+\Delta\phi) \\ \sin\Delta\phi.\cos\Delta\theta \end{bmatrix} \tag{63}$$

Making the substitutions:

$$\left. \begin{aligned} \eta &= \mu.\cos\beta \\ \\ \psi &= \mu.\sin\beta \\ \\ \mu &= (\eta^2+ \psi^2)^{\frac{1}{2}} \end{aligned} \right\} \tag{64}$$

and including terms of second order in η and ψ from relation (1), the right hand side of equation (62) becomes:

$$\begin{bmatrix} \tfrac{1}{2}\mu^2.(\cos^2\phi.\cos^2(\theta-\beta) - 1) \\[2ex] \mu.\sin\phi.\cos(\theta-\beta) - \tfrac{1}{4}\mu^2.\cos\phi.(\sin2(\theta-\beta) + \sin2\beta) \\[2ex] -\mu.\sin(\theta-\beta) - \tfrac{1}{4}\mu^2.\sin2\phi.\cos^2(\theta-\beta) \end{bmatrix} \qquad (65)$$

Equating the second and third elements of (63) and (65), and using the approximations:

$$\cos(\phi+\Delta\phi) = \cos\phi/(1 + \tan\phi.\sin\Delta\phi)$$

$$\cos\Delta\theta = 1$$

we find approximate expressions for:

$$\left.\begin{array}{l} \sin\Delta\theta \simeq \mu.\tan\phi.\cos(\theta-\beta) - \tfrac{1}{4}\mu^2.\{(1+2\tan^2\phi).\sin2(\theta-\beta) + \sin2\beta\} \\[3ex] \sin\Delta\phi \simeq -\mu.\sin(\theta-\beta) - \tfrac{1}{4}\mu^2.\sin2\phi.\cos^2(\theta-\beta) \end{array}\right\} \quad (66)$$

12.2 Maximum value of boresight errors

For a fixed value of μ, and for positive values of ϕ, we find, by differentiation with respect to θ, and subsequent inspection, that:

(a) $\sin\Delta\phi$ has its maximum positive value when:

$$(\theta-\beta) = -\pi/2$$

$$\beta = \text{any value}$$

(b) $\sin\Delta\theta$ has its maximum positive value when:

$$\beta = -\pi/4 \quad \text{or} \quad 3\pi/4$$

$$\sin(\theta-\beta) = -\frac{\tan\phi}{2\mu.(1+2\tan^2\phi)} \cdot \left\{ \left(1 + \frac{2\mu^2.(1+2\tan^2\phi)^2}{\tan^2\phi} \right)^{\frac{1}{2}} - 1 \right\} \qquad (67)$$

Figure 5 shows $\Delta\theta$ and $\Delta\phi$ plotted as functions of μ, for various values of ϕ, and with these values of β and of $(\theta-\beta)$. The latter is also plotted in the figure.

Note that:

(1) β is the angle of the resultant axis of tilt, due to the combined

action of η and ψ.

(2) for a fixed value of $(\eta^2+\psi^2)$, the maximum value of $\Delta\theta$ occurs when η and ψ are equal in magnitude; that is $\beta = -\pi/4$ or $3\pi/4$.

(3) for a fixed value of $(\eta^2+\psi^2)$, the maximum value of $\Delta\phi$ is $(\eta^2+\psi^2)^{\frac{1}{2}}$, and is independent of β or ϕ.

(4) the maximum value of $\Delta\phi$ occurs when $\theta = \beta-\pi/2$; that is, the boresight is normal to the effective tilt axis.

(5) the maximum value of $\Delta\theta$ occurs when θ is in the range $(\beta$ to $(\beta-\pi/4))$, depending on the values of μ and ϕ (figure 5(b)).

(6) if the tilt of the azimuth axis is due to a single spiral deflection, then $\mu = A = \eta_{max} = \psi_{max}$.

(7) if the pitch and roll errors are uncorrelated sinusoidal motions, then ψ_{max} and η_{max} can occur simultaneously and the value of μ is increased by $2^{\frac{1}{2}}$; that is $\mu = 2^{\frac{1}{2}}.\eta_{max} = 2^{\frac{1}{2}}.\psi_{max}$.

(8) $\Delta\theta$ is not zero when ϕ is zero, due to second order effects.

13. EFFECT OF TILTED ELEVATION AXIS

13.1 General

We wish to examine the first order effect of incorrect alignment of the elevation axis on the boresight errors and on the motor axis torques.

If 0_e, the origin of the e-system coordinates fixed to the elevating assembly, is displaced from its nominal position, the effect is equivalent to a variation in the coordinates, X_1, Z_1 and θ, and can be evaluated accordingly.

In the remainder of this Section, we consider the misalignment in the direction of the elevation axis, $0_e Y_e$.

13.2 Misalignment matrix

Suppose the elevation axis, $0_e Y_e$, is rotated through the small angle, $D\theta$, about the normal through 0_e, parallel to $0Z_r$; followed by rotation through the small angle, $D\eta$, about the normal through 0_e, parallel to the $X_r Y_r$-plane. This is illustrated in figures 1(d) and 6. Then, to a first order approximation in $D\theta$ and $D\eta$:

$$P_r = E + M.D.P_e$$

$$M = \begin{bmatrix} 1 & -D\theta & 0 \\ D\theta & 1 & -D\eta \\ 0 & D\eta & 1 \end{bmatrix} \qquad (68)$$

Subsequent analysis is based on the assumption that roll and pitch correction errors are also small. In this case, to a first order approximation:

$$A.B = I$$

$$P_b = P_s = C.(E + M.D.P_e)$$

$$(69)$$

13.3 Boresight error

Using the same argument as for Section 12.1, it follows that:

$$(C+\Delta C).(E + M.(D+\Delta D).(r+\Delta r).Z) = C.(E + D.r.Z) \qquad (70)$$

Neglecting $D^{-1}.E/r$ and $\Delta r/r$, as in Section 12.1, and multiplying on the left by $D^{-1}.C^{-1}$:

$$(I + D^{-1}.C^{-1}.\Delta C.D).(I + D^{-1}.(M-I).D).(I + D^{-1}.\Delta D).Z \simeq Z$$

Neglecting second order product terms between ΔC, $(M-I)$ and ΔD:

$$(D^{-1}.C^{-1}.\Delta C.D + D^{-1}.(M-I).D + D^{-1}.\Delta D).Z \simeq 0$$

By evaluation:

$$\sin\Delta\phi \simeq 0$$

$$\sin\Delta\theta \simeq D\eta.\tan\phi - D\theta$$

$$(71)$$

13.4 Elevation torque changes

Changes in elevation torque due to the inclusion of the misalignment matrix, follow from equation (14):

$$\Delta T_e \simeq -\Lambda(R^{-1}.R).$$

$$- \quad R^{-1}.\dot{R} + \frac{d}{dt} \qquad \qquad \Delta(R^{-1}.\dot{R}).P$$

$$- \sum_e m.N(P).D^{-1}.D.\Delta(R^{-1}.(G + \ddot{L})) \qquad (72)$$

Key results in the evaluation are given in items (a) to (e) of Appendix II, and the final results are listed in the left hand side of Table 7. Note that the y-component is the change in torque required of the elevation servo motor.

13.5 Pitch, roll and azimuth torque changes

Using equation (16) to determine the torque changes in the r-system:

$$\Delta T_r \simeq \Delta(Q).T_e + Q.\Delta T_e - \sum_e m.N(K).\Delta(Q.R^{-1}).G$$

$$- \sum_e m.N(K).\Delta(Q.R^{-1}.\ddot{P}^{\prime}) \qquad (73)$$

Key results in this evaluation are given in items (f) to (k) of Appendix II. Final results are listed in the right hand side of Table 7. Note that the z-component is the change in torque required of the azimuth servo motor.

Equation (20) is used to derive $\Delta(T_{roll}+j.T_{pitch})$ from ΔT_r, and this is listed in Table 8.

13.6 Discussion of results

From equation (71), both $D\eta$ and $D\theta$ contribute a significant error in azimuth boresight, but the contribution of $D\theta$ is constant, and can be calibrated out.

From Tables 8 and 5, most changes in roll and pitch torques are small compared with components existing with no axis tilt. The only changes not so swamped, are the terms in $D\eta.(J_e - Q_e)$ in the first two columns of Table 8. These may be of significant magnitude in some applications.

A similar comparison between Tables 7, 3 and 2, shows that none of the changes in azimuth and elevation torque are swamped by non-perturbed torques. The most significant terms are those in $2D\eta.J_e$ coupled with $\ddot{\theta}$ and $\ddot{\phi}$.

Thus $D\eta$ is more significant in its effects than $D\theta$, and requires closer control.

14. EFFECT OF NON-INTERSECTION OF AZIMUTH, PITCH AND ROLL AXES

14.1 Misalignment matrix

We consider the case of translation (without rotation) of the pitch correction, and azimuth axes so that they do not intersect each other, or the roll correction axis.

Figure 7(a) shows the pitch correction axis, $O_g Y_g$, in a plane normal to the roll correction axis, $O_b X_b$. $O_g O_b$ is normal to both axes, and is of length, Z_2. The coordinates of a single point, P, are related by:

$$P_b \;=\; A.(P_g + [0 \; 0 \; Z_2]^T)$$

Figure 7(b) shows the azimuth axis, $O_s Z_s$, in a plane normal to the pitch correction axis, $O_g Y_g$. $O_s O_g'$ is normal to both axes, is of length, X_2, and is collinear with the axis, $O_s X_s$. $O_g' O_g$ is of length, Y_2. Then:

$$P_g \;=\; B.(P_s + [X_2 \; Y_2 \; 0]^T)$$

Combining these relations with the approximation of equation (13):

$$
\left.
\begin{aligned}
P_b &\simeq (I + X).(P_s + E_2) - Y.E_2 \\[2mm]
E_2 &= [X_2 \; Y_2 \; Z_2]^T \\[2mm]
Y &= \begin{bmatrix} 0 & 0 & \psi \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
\right\} \tag{74}
$$

14.2 Boresight error

Using the method of Sub-section 12.1:

$$A.B.(C+\Delta C).(E + (D+\Delta D).(r+\Delta r).Z) + (A.B-Y).E_2 \;=\; A.B.C.(E + D.r.Z.) \tag{75}$$

After multiplication by $B^{-1}.A^{-1}/r$, and neglecting terms of second order of smallness:

$$\Delta C.D.Z + C.\Delta D.Z \simeq -(I-Y).E_2/r \tag{76}$$

Thus for small values of the elements of E_2 (compared with r, the distance to the target), the boresight error is negligible.

14.3 Torque changes in the s-system

Using relations (10) and (77) to determine the change in T_s due to a non-zero value of E_2:

$$
\left.
\begin{aligned}
P &= P_s = C.P_r \\[2ex]
P' &= P_b = (I+X).P_s + (I+X-Y).E_2 \\[2ex]
R &= I + X \\[2ex]
L &= (I+X-Y).E_2
\end{aligned}
\right\} \tag{77}
$$

and:

$$
\begin{aligned}
\Delta T_s &= - \sum_{r,e} m.N(P).R^{-1}.\ddot{L} \\[2ex]
&\simeq -N(C.\sum_{r,e} m.P_r).(\ddot{X}-\ddot{Y}).E_2
\end{aligned} \tag{78}
$$

Clearly, all terms involve $\ddot{\eta}$ or $\ddot{\psi}$, and none involve time-derivatives of ϕ. Key, and final, results are given in items (a) to (c) of Appendix III. The z-component of ΔT_s is the change in torque required of the azimuth motor.

14.4 Torque changes about the elevation axis

Using the relations (12) and (79) to determine the change in the y-component of T_e due to the non-zero value of E_2:

$$P = P_s = C.(E + D.P_e)$$

$$P_e = D^{-1}.C^{-1}.P_s - D^{-1}.E$$

$$Q = D^{-1}.C^{-1} \tag{79}$$

$$K = - D^{-1}.E$$

$$P',R \text{ from equation (77)}$$

and:

$$\Delta T_e = Q.\Delta T_s - \sum_e m.N(K).Q.R^{-1}.\frac{d^2}{dt^2}\{(I+X-Y).E_2\}$$

$$\simeq D^{-1}.C^{-1}.\Delta T_s + M_e.D^{-1}.N(E).C^{-1}.(I-X).(\ddot{X}-\ddot{Y}).E_2$$

$$\simeq D^{-1}.C^{-1}.(\Delta T_s + M_e.N(C.E).(\ddot{X}-\ddot{Y}).E_2) \tag{80}$$

Again, all terms involve $\ddot{\eta}$ or $\ddot{\psi}$, and none involve time-derivatives of ϕ. The y-component of ΔT_e is the change in torque required of the elevation motor. Key, and final, results in its evaluation are given in items (d) to (g) of Appendix III.

14.5 Torque changes in roll and pitch

Using relations (12) and (81):

$$Q_b = A.B = (I+X)$$

$$K_b = A.[0 \ 0 \ Z_2]^T + A.B.[X_2 \ Y_2 \ 0]^T$$

$$\simeq A.B.[(X_2-\psi.Z_2) \ Y_2 \ Z_2]^T \tag{81}$$

$$P' = P_b = (I+X).P_s + K_b$$

then:

$$\Delta T_b = (I+X).\Delta T_s - \sum_{r,e} m.N(K_b).\left(G + \ddot{K}_b + \frac{d^2}{dt^2}\left\{(I+X).P_s\right\}\right)$$

$$\simeq \Delta T_s - N(K_b).M.G - N(E_2).\left\{ M.\ddot{K}_b + \sum_{r,e} m.(2\dot{X}.\dot{P}_s + \ddot{X}.P_s)\right\}$$

$$- (I+X).N\{(I-X).K_b\}.\sum_{r,e} m.\ddot{P}_s \tag{82}$$

Similarly, using relations (12) and (83):

$$\left.\begin{aligned} Q_g &= B \\[2mm] K_g &= B.[X_2 \ Y_2 \ 0]^T \\[2mm] P' &= P_b = (I+X).P_s + K_b \end{aligned}\right\} \tag{83}$$

then:

$$\Delta T_g = B.\Delta T_s - \sum_{r,e} m.N(K_g).\left(G + \ddot{K}_b + \frac{d^2}{dt^2}\left\{(I+X).P_s\right\}\right)$$

$$\simeq \Delta T_s - N(K_g).M.G - N(B^{-1}.K_g).\left\{ M.\ddot{K}_b + \sum_{r,e} m.(2\dot{X}.\dot{P}_s + \ddot{X}.P_s)\right\}$$

$$- (I+X).N\{(I-X).K_g\}.\sum_{r,e} m.\ddot{P}_s \tag{84}$$

The x-component of ΔT_b, and the y-component of ΔT_g, are the roll and pitch axis torque variations. Key results of the evaluation of equations (82) and (84) are given in items (h) to (q) of Appendix III. Table 9 lists the results (as well as those for azimuth and elevation, derived previously) except that terms in η and ψ have been ignored wherever they are swamped by larger terms in the same cell.

14.6 Discussion of torque changes

From Table 1 and relations (22) and (25) we see that H/M, V/M and H_e/M are small distances in most applications. Also, from equation (31):

$$|\ddot{\eta}/\dot{\theta}^2|_{max} \simeq |\ddot{\psi}/\dot{\theta}^2|_{max} \simeq (\omega^2/\dot{\theta}^2).\eta_{max}$$

$$|\dot{\eta}/\dot{\theta}|_{max} \simeq |\dot{\psi}/\dot{\theta}|_{max} \simeq (\omega/\dot{\theta}).\eta_{max}$$

are small quantities in most applications. Hence the terms in Table 9 can be divided into groups of first, second and third order of smallness.

Only the terms in g are of first order and these will probably determine the maximum values of X_2 and Y_2. Those terms independent of η and ψ, together with the elevation terms in M_e, comprise the second order group. These may be significant. The remaining (third order) terms are most likely negligible for small values of η_{max}.

15. NON-NORMALITY OF PITCH AND ROLL AXES

15.1 Misalignment matrix

Consider the Cartesian coordinate system, suffixed g', derived by rotation of the g-system (fixed to the gimbal ring) through the small angle, $D\theta$, about the common z-axis. Then:

$$\left.\begin{array}{c} P_g = M.P_g{}' \\[2em] M \simeq \begin{bmatrix} 1 & -D\theta & 0 \\ D\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}\right\} \quad (85)$$

Suppose the pitch-correction axis is now transferred from the y-axis of the g-system, to the y-axis of the g'-system. The pitch and roll axes are still coplanar, and intersect at a common point, but this intersection departs from normal by the small angle, $D\theta$. Clearly now:

$$P_b = A.M.B.P_s \quad (86)$$

and A.B is replaced by A.M.B wherever it occurs.

By evaluation, neglecting terms of second order of smallness:

$$A.M.B \simeq \begin{bmatrix} 1 & -D\theta & \psi \\ D\theta & 1 & \eta \\ -\psi & \eta & 1 \end{bmatrix} \quad (87)$$

Similarly, by evaluation, we find that:

$$A.M.B.C \simeq (A.B.C)' \tag{88}$$

where the slash denotes replacement of θ by $(\theta+D\theta)$ wherever it occurs.

15.2 Boresight error

Following the argument discussed in Sub-section 12.1, it is clear that:

$$A.M.B.(C+\Delta C).(D+\Delta D).Z = A.B.C.D.Z$$

if:

$$\left.\begin{array}{rcl} \Delta\theta & \simeq & -D\theta \\[2em] \Delta\phi & \simeq & 0 \end{array}\right\} \tag{89}$$

Because $\Delta\theta$ is constant it can be calibrated out.

15.3 Torque changes

It follows from relations (15), (17) and (18) that replacement of (A.B.C) by $(A.B.C)'$ (ie replacement of θ by $(\theta+D\theta)$ wherever it occurs) accounts for the changes in T_e, T_r, T_{elev} and T_{az}.

The changes in roll and pitch axis torques follow from:

$$T_{roll} = T_{gx} = [1\ 0\ 0].M.B.C.T_r' \simeq T_{roll}' \tag{90}$$

$$\begin{aligned} T_{pitch} = T_{sy} &= [0\ 1\ 0].C.T_r' \\[1em] &= [0\ 1\ 0].(I + (M^{-1}-I)).M.C.T_r' \\[1em] &\simeq T_{pitch}' - D\theta.[1\ 0\ 0].M.B.C.T_r' \\[1em] &\simeq T_{pitch}' - D\theta.T_{roll}' \tag{91} \end{aligned}$$

15.4 Discussion

Thus the effect of the non-normality is accounted for by replacement of θ by $(\theta+D\theta)$ in all boresight error and motor torque relations. In addition, the pitch axis receives a component of the roll axis torque, proportional to the angular deviation from normality. This crosscoupling of torque is the only significant effect of the non-normality, and cannot be eliminated by calibration of θ.

TABLE 7. CHANGES IN T_e AND T_r DUE TO TILTED ELEVATION AXIS

TABLE 6.   COMBINED VALUES OF $(T_{roll}+j.T_{pitch})$ AT FIXED ELEVATION

	$\ddot{\theta}$	$\dot{\theta}^2$	$\dfrac{\ddot{\eta}+j\ddot{\psi}}{2}$	$\dfrac{\ddot{\eta}-j\ddot{\psi}}{2}$	$\dfrac{\dot{\eta}+j\dot{\psi}}{2}.\dot{\theta}$	$\dfrac{\dot{\eta}-j\dot{\psi}}{2}.\dot{\theta}$	g
J	-2ψ		-2		$4j$		
Q			-2				
K				$-2e^{j2\theta}k$		$-4je^{j2\theta}k$	
L	$e^{j\theta}L$	$je^{j\theta}L$					
H							$je^{j\theta}h$
V							$\eta+j\psi$

TABLE 5.   CONTRIBUTION OF ELEVATING ASSEMBLY TO $(T_{roll}+j.T_{pitch})$

	$\dot\theta$	$\dot\theta^2$	$\ddot\phi$	$\dot\phi^2$	$\ddot\theta.\dot\phi$	$\dfrac{\ddot\eta+j\ddot\upsilon}{2}$	$\dfrac{\eta-j\upsilon}{2}$	$\dfrac{\eta+j\upsilon}{2}.\dot\theta$	$\dfrac{\eta-j\upsilon}{2}.\dot\theta$	$\dfrac{\eta+j\upsilon}{2}.\dot\phi$	$\dfrac{\eta-j\upsilon}{2}.\dot\phi$	ϵ
J_e	$-\dot\upsilon$		$2je^{j\theta}$		$-2e^{j\theta}$	-5	$e^{j2\theta}$	$2j$	$2je^{j2\theta}$			
Q_e	$-\dot\upsilon$					-1	$-e^{j2\theta}$	$2j$	$-2je^{j2\theta}$			
$K_e.\cos2\phi_k$	$\dot\upsilon$				$-2e^{j\theta}$	-1	$-e^{j2\theta}$	$-2j$	$-2je^{j2\theta}$		$2e^{j2\theta}$	$je^{j\theta}$
$K_e.\sin2\phi_k$	$-e^{j\theta}$	$-je^{j\theta}$	$-e^{j\theta}$	$-\dot\upsilon$	$-2\dot\upsilon$							$\eta+j\upsilon$
$L_e.\cos\phi_L$							$2je^{j2\theta}$		$-4e^{j2\theta}$		$-2je^{j2\theta}$	$-e^{j\theta}$
$L_e.\sin\phi_L$	$je^{j\theta}$	$-e^{j\theta}$		$e^{j\theta}$		$-2x_1$	$2x_1e^{j2\theta}$	$4jx_1$	$4jx_1.e^{j2\theta}$	$-4\bar z_i$	$-2je^{j2\theta}$	$je^{j\theta}$
$H_e.\cos\phi_h$	$\bar z_i.e^{j\theta}-2x_1.\dot\upsilon$	$j\bar z_i.e^{j\theta}$	$jx_1.e^{j\theta}$	$j\bar z_i.e^{j\theta}$		$-2x_1$		$4jx_1$	$4jx_1.e^{j2\theta}$	$-4\bar z_i$		$\eta+j\upsilon$
$H_e.\sin\phi_h$	$x_1.e^{j\theta}$	$jx_1.e^{j\theta}$	$j\bar z_i.e^{j\theta}$	$-jx_1.e^{j\theta}$	$-2\bar z_i.e^{j\theta}+2x_1.\dot\upsilon$	$-4\bar z_i$			$2x_1$	$2x_1$	$-2x_1.e^{j2\theta}$	$-e^{j\theta}$
V_e	$j\bar z_i.e^{j\theta}$	$-\bar z_i.e^{j\theta}$					$2jx_1.e^{j2\theta}$	$2jx_1^2$	$-4x_1.e^{j2\theta}$	$2x_1$		$-e^{j\theta}$
M_e	$x_1.\bar z_i.e^{j\theta}-x_1^2.\dot\upsilon$	$jx_1.\bar z_i.e^{j\theta}$				$-x_1^2-2\bar z_i^2$	$x_1^2.e^{j2\theta}$	$2jx_1^2$	$2jx_1^2.e^{j2\theta}$			$jx_1.e^{j\theta}+\bar z_i(\eta+j\upsilon)$

TABLE 4. CONTRIBUTION OF ROTATING ASSEMBLY TO T_r

		$\ddot{\theta}$	$\dot{\theta}^2$	$\frac{\ddot{\eta}\pm j\ddot{\psi}}{2}.e^{\mp j\theta}$	$\frac{\dot{\eta}\pm j\dot{\psi}}{2}.\dot{\theta}.e^{\mp j\theta}$	g	$\frac{\eta\pm j\psi}{2}.g.e^{\mp j\theta}$
x	J_r			-1	$\pm 2j$		
	Q_r			-1			
	K_r			$-e^{\pm j\beta_r}$	$\pm 2je^{\pm j\beta_r}$		
	L_r	$\cos\gamma_r$	$-\sin\gamma_r$				
	H_r					$-\sin a_r$	
	V_r						1
	M_e	$X_1.Z_1$		$-Z_1^2$			Z_1
y	J_r			$\pm j$	2		
	Q_r			$\pm j$			
	K_r			$\mp je^{\pm j\beta_r}$	$-2e^{\pm j\beta_r}$		
	L_r	$\sin\gamma_r$	$\cos\gamma_r$				
	H_r					$\cos a_r$	
	V_r						$\mp j$
	M_e		$X_1.Z_1$	$\pm j(X_1^2+Z_1^2)$	$2X_1^2$	X_1	$\mp jZ_1$
z	J_r	-2		$.$			
	Q_r						
	K_r						
	L_r			$e^{\mp j\gamma_r}$			
	H_r						$-e^{\mp ja_r}$
	V_r						
	M_e	$-X_1^2$		$X_1.Z_1$			$-X_1$

TABLE 5.   CONTRIBUTION OF ELEVATING ASSEMBLY TO T_r

		θ	$\dot\theta$	$\ddot\theta$	$\theta.\phi$	$\eta^{+}.\dot\theta^{2}.\cos^{?}\theta$	$\eta^{+}.\dot\theta^{2}.\theta.\cos^{?}\theta$	$\eta^{+}.\dot\theta^{2}.\phi.\cos^{?}\theta$	ε	$\eta^{+}.\dot\phi^{2}.\cos^{?}\theta$
x	J_e	$-\sin 2\phi_k$		$-2\cos 2\phi_k$	-2	-1	$+2j$			
	Q_e					-1				
	K_e	$X_1.\sin\phi_h$ $+z_1.\cos\phi_h$	$-\sin 2\phi_k$	$-z_1.\sin\phi_h$		$-\cos 2\phi_k$	$-2\cos\phi_L$	$\pm\sin 2\phi_k$		$\sin\phi_h$
	L_e	$\sin\phi_L$	$-\cos\phi_L$	$\sin\phi_L$		$\pm j\cos\phi_L$		$\mp 2j\sin\phi_L$		
	H_e		$X_1.\sin\phi_h$ $+z_1.\cos\phi_h$	$-z_1.\sin\phi_h$	$-z_1.\sin\phi_h$	$-z_1.\sin\phi_h$	$-2X_1$	$-z_1.\cos\phi_h$		
	V_e	$\pm j$	$\pm j$			$\pm jX_1$				
y	J_e	-1				$\pm 2j$	2		-1	
	Q_e	-1	$-\cos\phi_L$	2						
	K_e		$-\sin 2\phi_k$				$-2\cos 2\phi_k$			
	L_e	$\sin\phi_L$				$\cos\phi_L$	$\pm 2j\cos\phi_L$			
	H_e		$X_1.\cos\phi_h$ $+z_1.\sin\phi_h$	$-X_1.\sin\phi_h$ $+z_1.\cos\phi_h$		$\pm 2j(X_1.\cos\phi_h+z_1.\sin\phi_h)$	$4X_1.\cos\phi_h$	$\pm 2j(z_1.\cos\phi_h-Y_1.\sin\phi_h)$	$\cos\phi_h$	$\mp j\sin\phi_h$
	V_e	$\mp j$				X_1	$\mp 2jX_1$	2		
z	J_e	-1								
	Q_e	-1								
	K_e	$\cos 2\phi_k$	$-\sin 2\phi_k$	$-2\sin 2\phi_k$	$-\sin 2\phi_k$	$-\sin 2\phi_k$		$-2\cos 2\phi_k$		
	L_e		$-\cos\phi_L$			$\mp j\sin\phi_L$		$\mp 2j\cos\phi_L$		
	H_e	$-2X_1.\cos\phi_h$		$2X_1.\sin\phi_h$		$X_1.\sin\phi_h+z_1.\cos\phi_h$		$2X_1.\cos\phi_h$		$-\cos\phi_h$
	V_e					$\mp jz_1$				$\pm j$

TABLE 2.   CONTRIBUTION OF ELEVATING ASSEMBLY TO T_e

		$\dot\theta$	$\dot\theta^2$	$\dot\phi$	$\dot\phi^2$	$\dot\theta.\dot\phi$	$\frac{\eta\pm j\ddot\psi}{2}.e^{\mp j\theta}$	$\frac{\eta\pm j\dot\psi}{2}.\dot\theta.e^{\mp j\theta}$	$\frac{\eta\pm j\dot\psi}{2}.\dot\phi.e^{\mp j\theta}$	g	$\frac{\eta\pm j\dot\psi}{2}.g.e^{\mp j\theta}$
x	J_e	$-\sin\phi$				$-2\cos\phi$	$-\cos\phi$		$2\sin\phi$		
	Q_e	$-\sin\phi$					$-\cos\phi$	$\pm 2j\cos\phi$			
	K_e	$-\sin(\phi-\beta_e)$				$-2\cos(\phi-\beta_e)$	$-\cos(\phi-\beta_e)$		$2\sin(\phi-\beta_e)$		
	L_e		$-\sin\phi_L.\cos\phi$	$-\cos\gamma_e$	$\sin\gamma_e$		$\mp j\cos\gamma_e$	$-2\cos\phi_L.\cos\phi$	$\pm 2j\sin\gamma_e$		
	H_e	$X_1.\sin\alpha_e$					$-z_1.\sin\alpha_e$				$\sin\alpha_e$
	V_e		$-X_1.\sin\phi$				$\mp j(z_1.\sin\phi+X_1.\cos\phi)$	$-2X_1.\cos\phi$		$-\cos\phi$	$\pm j\sin\phi$
y	J_e			2			$\pm 2j$	2			
	Q_e										
	K_e		$-\sin2\phi_k$					$-2\cos2\phi_k$			
	L_e	$\sin\phi_L$					$\cos\phi_L$	$\mp 2j.\cos\phi_L$			
	H_e		$X_1.\sin\phi_h$				$\pm j(z_1.\sin\phi_h+X_1.\cos\phi_h)$	$2X_1.\cos\phi_h$		$\cos\phi_h$	$\mp j\sin\phi_h$
	V_e										
z	J_e	$-\cos\phi$				$2\sin\phi$	$\sin\phi$		$2\cos\phi$		
	Q_e	$-\cos\phi$					$\sin\phi$	$\mp 2j\sin\phi$			
	K_e	$\cos(\phi-\beta_e)$				$-2\sin(\phi-\beta_e)$	$-\sin(\phi-\beta_e)$		$-2\cos(\phi-\beta_e)$		
	L_e		$\sin\phi_L.\sin\phi$	$-\sin\gamma_e$	$-\cos\gamma_e$		$\mp j\sin\gamma_e$	$2\cos\phi_L.\sin\phi$	$\mp 2j\cos\gamma_e$		
	H_e	$-X_1.\cos\alpha_e$					$z_1.\cos\alpha_e$				$-\cos\alpha_e$
	V_e		$-X_1.\cos\phi$				$\pm j(X_1.\sin\phi-z_1.\cos\phi)$	$2X_1.\sin\phi$		$\sin\phi$	$\pm j\cos\phi$

TABLE 1. MASS DISTRIBUTION PARAMETERS

Combined

$$M = \sum_{r,e} m = M_r + M_e$$

$$H.\cos\alpha = \sum_{r,e} m.x_r = H_r.\cos\alpha_r + M_e.X_1 + H_e.\cos\phi_h$$

$$H.\sin\alpha = \sum_{r,e} m.y_r = H_r.\sin\alpha_r + V_e$$

$$V = \sum_{r,e} m.z_r = V_r + M_e.z_1 + H_e.\sin\phi_h$$

$$Q = \sum_{r,e} m.z_r^2 = Q_r + M_e.z_1^2 + J_e + K_e.\cos2\phi_k + 2z_1.H_e.\sin\phi_h$$

$$J = \sum_{r,e} m.\frac{x_r^2+y_r^2}{2} = J_r + \frac{1}{2}(J_e+Q_e+M_e.X_1^2-K_e.\cos2\phi_k) + X_1.H_e.\cos\phi_h$$

$$K.\cos\beta = \sum_{r,e} m.\frac{y_r^2-x_r^2}{2} = K_r.\cos\beta_r + \frac{1}{2}(-J_e+Q_e-M_e.X_1^2+K_e.\cos2\phi_k) - X_1.H_e.\cos\phi_h$$

$$K.\sin\beta = \sum_{r,e} m.x_r.y_r = K_r.\sin\beta_r + X_1.V_e + L_e.\cos\phi_l$$

$$L.\cos\gamma = \sum_{r,e} m.x_r.z_r = L_r.\cos\gamma_r + M_e.X_1.z_1 + H_e.(X_1.\sin\phi_h+z_1.\cos\phi_h) - K_e.\sin2\phi_k$$

$$L.\sin\gamma = \sum_{r,e} m.y_r.z_r = L_r.\sin\gamma_r + z_1.V_e + L_e.\sin\phi_l$$

$$\phi_h = \phi + \alpha_e \qquad \theta_h = \theta + \alpha$$
$$2\phi_k = 2\phi - \beta_c \qquad 2\theta_k = 2\theta - \beta$$
$$\phi_l = \phi + \gamma_e \qquad \theta_l = \theta + \gamma$$

r-component

$$M_r = \sum_r m$$

$$H_r.\cos\alpha_r = \sum_r m.x_r$$

$$H_r.\sin\alpha_r = \sum_r m.y_r$$

$$V_r = \sum_r m.z_r$$

$$Q_r = \sum_r m.z_r^2$$

$$J_r = \sum_r m.\frac{x_r^2+y_r^2}{2}$$

$$K_r.\cos\beta_r = \sum_r m.\frac{y_r^2-x_r^2}{2}$$

$$K_r.\sin\beta_r = \sum_r m.x_r.y_r$$

$$L_r.\cos\gamma_r = \sum_r m.x_r.y_r$$

$$L_r.\sin\gamma_r = \sum_r m.y_r.z_r$$

e-component

$$M_e = \sum_e m$$

$$H_e.\cos\alpha_e = \sum_e m.x_e$$

$$H_e.\sin\alpha_e = \sum_e m.z_e$$

$$V_e = \sum_e m.y_e$$

$$Q_e = \sum_e m.z_e^2$$

$$J_e = \sum_e m.\frac{x_e^2+z_e^2}{2}$$

$$K_e.\cos\beta_e = \sum_e m.\frac{z_e^2-x_e^2}{2}$$

$$K_e.\sin\beta_e = \sum_e m.x_e.z_e$$

$$L_e.\cos\gamma_e = \sum_e m.x_e.y_e$$

$$L_e.\sin\gamma_e = \sum_e m.y_e.z_e$$

Symbol	Section First Used	Defined	Remarks
α	4	T1	angle associated with H
α	6	E31	log. increment of spiral motion
β	4	T1	angle associated with K
β	12	E64	angle associated with μ
γ	4	T1	angle associated with L
η	1	-	roll error angle
θ	1	-	azimuth rotation angle
μ	12	E64	amplitude of spiral motion
ϕ	1	-	elevation angle
ψ	1	-	pitch error angle
ω	6	E31	angular frequency of spiral motion
ω_0	6	E29	natural frequency of roll and pitch servo
$*$	4	-	complex conjugate operator
$'$	3	-	indicates inertial coordinate system
$'$	4	-	indicates replacement of θ by $(\theta+\pi)$
$'$	15	-	indicates replacement of θ by $(\theta+D\theta)$
$''$	3	-	indicates transformed coordinate system

Symbol	Section First Used	Defined	Remarks
c_0	6	-	servo damping factor
d	6	E29	ratio of inertias
e	2	-	suffix for e-coordinate-system
f	6	E27	damping unbalance factor
f	8	-	suffix denoting forcing
f	8	E42	servo unbalance function
g	2	-	suffix for g-coordinate-system
g	3	-	acceleration of gravity
h	4	T1	suffix for reference to H
k	4	T1	suffix for reference to K
l	4	T1	suffix for reference to L
m	3	-	mass of point quantity
r	2	-	suffix for r-coordinate-system
r	12	-	range to target
s	2	-	suffix for s-coordinate-system
s	6	E31	complex variable
t	6	-	time
u	6	E27	servo stiffness unbalance factor
x	3	-	Cartesian coordinate
y	3	-	Cartesian coordinate
z	3	-	Cartesian coordinate
Δ	12	-	operator specifying a change in argument
$\Delta\theta$	12	-	azimuth error
$\Delta\phi$	12	-	elevation error
Σ_e	4	-	summation operator over elevating assembly
Σ_r	4	-	summation over rotating assembly
$\Sigma_{r,e}$	4	-	summation over rotating and all elevating assemblies

Symbol	Section First Used	Defined	Remarks
O	2	-	origin of Cartesian axes
P	2	-	point coordinate vector
Q	3	E11	transformation matrix
Q	4	T1	mass distribution parameter
R	3	E5	transformation matrix
T	3	-	torque
T	2	-	superfix denoting matrix transposition
T_{az}	9	-	torque about azimuth axis
T_{elev}	8	-	torque about elevation axis
T_M	8	-	motor torque
T_{pitch}	4	-	torque about pitch axis
T_{roll}	4	-	torque about roll axis
U	6	E27	servo stiffness factor
V	4	T1	mass distribution parameter
X	2	-	Cartesian axis
X	4	E13	pitch-roll matrix approximation
X_1	2	-	elevation axis offset component
X_2	14	-	pitch-roll axis offset
Y	2	-	Cartesian axis
Y	14	E74	matrix approximation
Y_2	14	-	pitch-roll axis offset
Z	2	-	Cartesian axis
Z	12	-	$[1 \quad 0 \quad 0]^T$
Z_1	2	-	elevation axis offset component
Z_2	14	-	pitch-roll axis offset
b	2	-	suffix for b-coordinate-system

NOTATION

Symbol	Section First Used	Defined	Remarks
A	2	E1	roll correction matrix
A	6	E31	complex amplitude of spiral deflection
B	2	E1	pitch correction matrix
B	6	E32	complex amplitude of spiral deflection
C	2	E1	azimuth rotation matrix
D	2	E1	elevation matrix
$D\eta$	13	-	elevation axis deflection angle
$D\eta$	16	-	azimuth axis deflection angle
$D\theta$	13	-	elevation axis deflection angle
$D\theta$	15	-	deflection angle of pitch axis
E	2	E1	elevation offset vector
E_2	14	E74	pitch-roll offset vector
F	3	-	point mass force vector
F	6	E27	servo damping factor
G	3	E6	gravitation vector
G	6	E32	servo transfer function
H	4	T1	mass distribution parameter
I	4	-	unit matrix
J	4	T1	mass distribution parameter
K	3	E11	offset vector for transformation
K	4	T1	mass distribution parameter
L	3	E5	offset vector
L	4	T1	mass distribution parameter
M	4	T1	mass of structure
M	13	E68	elevation axis deflection matrix
M	15	E85	pitch-roll deflection matrix
M	16	E92	pitch-azimuth deflection matrix
N	3	E3	torque generation matrix

to back about the axis of azimuth rotation, has been examined. Clear rules are given for determining whether the torques about the azimuth, pitch and roll axes, due to mass unbalance parameters of the nominally identical structures, cancel each other, or are additive. For many of the parameters, cancellation occurs only if the two assemblies are elevated identically and simultaneously.

The results of the analysis, presented in tabular and graphical form, provide a firm basis for the design of the servo drives and of the procedure and facilities for achieving the necessary degree of mass balance.

The motion of the stabilised platform can be described in terms of a sequence of small amplitude spiral nutations of the azimuth axis about the true vertical direction.

Two of the main components consist of the transient decay of an initial deflection of the azimuth axis from vertical. One of these is in the forward direction (the same direction as the azimuth scanning rotation) and the other is in the reverse direction. They are mirror images, although executed at widely different rates. The actual angular velocity of the nutations depends on the stiffness and damping factor of the pitch and roll servo loops, the rate of azimuth rotation, and the distribution of mass of the combined rotating and elevating assemblies.

The third main component is a non-decaying (circular) nutation in the forward direction, and at the same rate as the azimuth scanning rotation. Its amplitude depends on the degree of unbalance in the distribution of mass, as well as on the servo stiffness and damping, and the azimuth rate.

Each of the three main components spawns two minor components proportional to its amplitude. One, at the same rate and opposite direction to the parent nutation, is proportional to the unbalance in the stiffness and damping of the pitch and roll servo loops. The other, at a rate and direction depending upon both that of the parent nutation and of the scanning rotation, is proportional to another measure of the unbalance in the mass distribution.

Each of the minor components, in turn, spawns two more minor components, ad infinitum, but the amplitudes of all these are negligible for modest amounts of unbalance.

Many imperfections of mass balance and axis geometry have an effect on the accuracy of the boresight direction, but most are negligibly small, or are cancelled out during the calibration process. Those that have a significant residual effect (and therefore require appropriate control) are:

(a) the instantaneous amplitude of the total nutation of the azimuth axis about the vertical

(b) rotation of the elevation axis from its nominal position, in the vertical plane

(c) non-normality of the pitch and azimuth axes.

$$T_{roll} = T_{gx} = [1\ 0\ 0].B.M.C.T_r'$$

$$= [1\ 0\ 0].A.B.M.C.T_r'$$

$$= T_{roll}' \tag{95}$$

$$T_{pitch} = T_{sy} = [0\ 1\ 0].M.C.T_r'$$

$$= [0\ 1\ -D\eta].C.T_r'$$

$$= T_{pitch}' - D\eta.T_{az}' \tag{96}$$

16.4 Discussion

Thus the effect of the non-normality is accounted for by replacing η by $(\eta+D\eta)$ in all boresight error and motor torque relations. In addition, the pitch axis receives a component of the azimuth axis torque proportional to the angular deviation from normality. This cross coupling of torque is the only significant effect of the non-normality, as it cannot be eliminated by the calibration of η.

17. CONCLUSION

The four-axis stabilised director system, while mounted in the normal erect position on a firm foundation, has been analysed to determine the driving torques, boresight errors, and motion of the stabilised platform caused by a small:

(a) deflection of the stabilised platform from the horizontal position

(b) unbalanced mass distribution on the rotating and elevating assemblies

(c) unbalance between the gain parameters of the roll and pitch correction servo loops

(d) deviations from the ideal geometrical relationships between the axes.

Techniques are presented for factorising the complex matrix relationships in such a way as to greatly simplify their evaluation.

The approximation of trigonometric functions of the roll and pitch angles by first order functions of the angles themselves (justified by the assumption that these angles are always small) greatly simplifies the interpretation of the results of the analysis by reducing significantly the number of terms contributing to each relationship. Conclusions drawn in respect of a particular application should be carefully checked for validity; if necessary using the accurate relationships. This is particularly important if the analysis were to be extended to the case of the director system mounted on a pitching and rolling vehicle.

The case of two separately elevatable assemblies, mounted symmetrically back

16. NON-NORMALITY OF PITCH AND AZIMUTH AXES

16.1 Misalignment matrix

Consider the Cartesian coordinate system, suffixed s', derived by rotation of the s-system (fixed to the stabilised platform) through the small angle, $D\eta$, about the common x-axis. Then:

$$P_s = M.P_s'$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -D\eta \\ 0 & D\eta & 1 \end{bmatrix} \tag{92}$$

Suppose the azimuth axis is now transferred from the z-axis of the s-system to the z-axis of the s'-system. Then the azimuth and pitch axes intersect at an angle departing from normality by the small angle, $D\eta$. Clearly now:

$$P_b = A.B.M.C.P_r \tag{93}$$

By evaluation, neglecting terms of second order of smallness:

$$A.B.M \simeq \begin{bmatrix} 1 & 0 & \psi \\ 0 & 1 & -(\eta+D\eta) \\ \psi & (\eta+D\eta) & 1 \end{bmatrix}$$

$$\simeq (A.B)' \tag{94}$$

where the slash denotes replacement of η by $(\eta+D\eta)$ wherever it occurs.

16.2 Boresight error

From equation (94), the effect of $D\eta$ is clearly the same as for the same magnitude of η, as determined in Sub-section 12.1.

16.3 Torque changes

It follows from relations (15), (17) and (18) that replacement of (A.B) by (A.B)' (ie replacement of η by $(\eta+D\eta)$ wherever it occurs) accounts for the changes in T_e, T_r, T_{elev} and T_{az}. The changes in roll and pitch axis torques follow from:

TABLE 8. CHANGES IN $(T_{roll} + j.T_{pitch})$ DUE TO TILTED ELEVATION AXIS

	$\ddot{\theta}.e^{j\theta}$	$\dot{\theta}^2.e^{j\theta}$	$\ddot{\phi}.e^{j\theta}$	$\dot{\phi}^2.e^{j\theta}$	$\dot{\theta}.\dot{\phi}.e^{j\theta}$	$g.e^{j\theta}$
J_e	$-jD\eta$	$D\eta$	$-2D\theta$		$-2jD\theta$	
Q_e	$jD\eta$	$-D\eta$				
K_e	$-jD\eta.\cos2\phi_k$ $-jD\theta.\sin2\phi_k$	$D\eta.\cos2\phi_k$ $+D\theta.\sin2\phi_k$			$2jD\eta.\sin2\phi_k$ $-2jD\theta.\cos2\phi_k$	
L_e	$D\eta.\cos\phi_L$ $-D\theta.\sin\phi_L$	$jD\eta.\cos\phi_L$ $-jD\theta.\sin\phi_L$	$jD\eta.\sin\phi_L$ $-jD\theta.\cos\phi_L$	$jD\eta.\cos\phi_L$ $+jD\theta.\sin\phi_L$	$-2D\eta.\sin\phi_L$	
H_e	$-jD\eta.Z_1.\sin\phi_h$ $+jD\theta.Z_1.\cos\phi_h$	$D\eta.Z_1.\sin\phi_h$ $-D\theta.Z_1.\cos\phi_h$	$-D\eta.Z_1.\cos\phi_h$ $-D\theta.Z_1.\sin\phi_h$	$D\eta.Z_1.\sin\phi_h$ $-D\theta.Z_1.\cos\phi_h$	$-2jD\eta.Z_1.\cos\phi_h$ $-2jD\theta.Z_1.\sin\phi_h$	$D\eta.\sin\phi_h$ $-D\theta.\cos\phi_h$
V_e	$D\eta.X_1$ $-D\theta.Z_1$	$jD\eta.X_1$ $-jD\theta.Z_1$				$-j.D\theta$

TABLE 9.   CHANGE IN TORQUES DUE TO NON-INTERSECTION OF AZIMUTH, PITCH AND ROLL AXES

		$\dot\theta$	$\dot\theta^2$	$\dot\phi$	$\dot\phi^2$	$\dot\theta.\dot\phi$	η	ν	$\eta.\dot\theta$	$\nu.\dot\theta$	$\eta.\dot\phi$	$\nu.\dot\phi$	
Roll	M	$z_2.\cos\theta_h$	$-z_2.\sin\theta_h$										
	H						$-(z_2^2+Y_2^2)$	$X_2.Y_2$	$-Y_2.\sin\theta_h$ $Y_2.\cos\theta_h$	$-2Y_2.\sin\theta_h$ $-2X_2.\cos\theta_h$			
	ν						$-z_2$						
	He			$-z_2.\sin\theta.$ $\sin\phi_h$	$-z_2.\sin\theta.$ $\cos\phi_h$	$-2z_2.\cos\theta.$ $\sin\phi_h$							$2\sin\phi_h.$ $(X_2.\sin\theta$ $-Y_2.\cos\theta)$
Pitch	M							$-X_2^2$					
	H	$\eta.X_2.\cos\theta_h$	$-\eta.X_2.\sin\theta_h$				$Y_2.\cos\theta_h$ $+X_2.\sin\theta_h$	$-2X_2.\cos\theta_h$	$2X_2.\cos\theta_h$	$2X_2.\sin\theta_h$		$-2X_2.\sin\theta.$ $\sin\phi_h$	$2X_2.\cos\theta.$ $\sin\phi_h$
	ν						$X_2.Y_2$						
	He			$-\eta.X_2.\sin\theta.$ $\sin\phi_h$	$-\eta.X_2.\sin\theta.$ $\cos\phi_h$	$-2\eta.X_2.\cos\theta.$ $\sin\phi_h$							
Azimuth	H						$z_2.\cos\theta_h$						
Elevation	H						$Y_2.\cos\alpha$	$-X_2.\cos\alpha$					
	ν						$z_2.\sin\theta.$						
	Me						$-z_1.z_2.\sin\theta$ $-X_1.Y_2$	$X_1.X_2$					

APPENDIX I

KEY RESULTS IN THE EVALUATION OF T_e AND T_r

Making the substitution:

$$a = \sin\theta$$

$$b = \cos\theta$$

$$c = \sin\phi$$

$$d = \cos\phi$$

(a) $C^{-1}.X.C =$
$$\begin{bmatrix} 0 & 0 & -(a.\eta-b.\psi) \\ 0 & 0 & -(b.\eta+a.\psi) \\ (a.\eta-b.\psi) & (b.\eta+a.\psi) & 0 \end{bmatrix}$$

$$= \tfrac{1}{2}(\eta+j.\psi).e^{-j\theta}.\begin{bmatrix} 0 & 0 & -j \\ 0 & 0 & -1 \\ j & 1 & 0 \end{bmatrix} + \tfrac{1}{2}(\eta-j.\psi).e^{j\theta}.\begin{bmatrix} 0 & 0 & j \\ 0 & 0 & -1 \\ -j & 1 & 0 \end{bmatrix}$$

Note that the second term is the complex conjugate of the first term.

(b) $C^{-1}.X.\dot{C} = \tfrac{1}{2}(\eta+j.\psi).e^{-j\theta}.\dot{\theta}.\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -j & 0 \end{bmatrix}$ + Conjugate

(c) $C^{-1}.\dot{C} = \dot{\theta}.\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $C^{-1}.\ddot{C} = \ddot{\theta}.\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \dot{\theta}^2.\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(e) $D^{-1}.\dot{D} = \dot{\phi}.\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(f) $R^{-1}.\dot{R} \quad \simeq \quad D^{-1}.C^{-1}.(I-X).\{\dot{X}.C.D + (I+X).(\dot{C}.D + C.\dot{D})\}$

$\simeq \quad D^{-1}.C^{-1}.\dot{X}.C.D + D^{-1}.C^{-1}.\dot{C}.D + D^{-1}.\dot{D}$

$\simeq \quad \tfrac{1}{2}(\dot{\eta}+j.\dot{\psi}).e^{-j\theta}.\begin{bmatrix} 0 & c & -j \\ -c & 0 & -d \\ j & d & 0 \end{bmatrix} + \text{Conjugate}$

$+ \dot{\theta}.\begin{bmatrix} 0 & -d & 0 \\ d & 0 & -c \\ 0 & c & 0 \end{bmatrix} + \dot{\phi}.\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(g) $\sum_{e} m.N(P).R^{-1}.\dot{R}.P \quad \simeq$

$\tfrac{1}{2}(\dot{\eta}+j.\dot{\psi}).e^{-j\theta}. \begin{bmatrix} (J_e+Q_e).d + K_e.\cos(\phi-\beta_e) + j.L_e.\cos\gamma_e \\ -L_e.\cos(\phi+\gamma_e) - 2j.J_e \\ -(J_e+Q_e).c + K_e.\sin(\phi-\beta_e) + j.L_e.\sin\gamma_e \end{bmatrix} + \text{Conjugate}$

$+ \dot{\theta}.\begin{bmatrix} (J_e+Q_e).c + K_e.\sin(\phi-\beta_e) \\ -L_e.\sin(\phi+\gamma_e) \\ (J_e+Q_e).d - K_e.\cos(\phi-\beta_e) \end{bmatrix} + \dot{\phi}.\begin{bmatrix} L_e.\cos\gamma_e \\ -2J_e \\ L_e.\sin\gamma_e \end{bmatrix}$

(h) $\sum_{e} m.N(P).D^{-1} = \begin{bmatrix} -c.V_e & -H_e.\sin\alpha_e & d.V_e \\ H_e.\sin(\phi+\alpha_e) & 0 & -H_e.\cos(\phi+\alpha_e) \\ -d.V_e & H_e.\cos\alpha_e & -c.V_e \end{bmatrix}$

(i) $\quad D.R^{-1}.G \quad = C^{-1}.(I-X).G$

$$= \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \tfrac{1}{2}(\eta+j.\psi).e^{-j\theta}.\begin{bmatrix} j.g \\ g \\ 0 \end{bmatrix} + \text{Conjugate}$$

(j) $\quad D.R^{-1}.\ddot{L} \quad = C^{-1}.(I-X).\{\ddot{X}.C.E + 2\dot{X}.\dot{C}.E + (I+X).\ddot{E}\}$

$$= \{C^{-1}.\ddot{X}.C + 2C^{-1}.\dot{X}.\dot{C} + C^{-1}.\ddot{C}\}.E$$

$$= \tfrac{1}{2}(\ddot{\eta}+j.\ddot{\psi}).e^{-j\theta}.\begin{bmatrix} -j.Z_1 \\ -Z_1 \\ j.X_1 \end{bmatrix} + \text{Conjugate}$$

$$+ (\dot{\eta}+j.\dot{\psi}).e^{-j\theta}.\dot{\theta}.\begin{bmatrix} 0 \\ 0 \\ X_1 \end{bmatrix} + \text{Conjugate}$$

$$+ \ddot{\theta}.\begin{bmatrix} 0 \\ X_1 \\ 0 \end{bmatrix} + \dot{\theta}^2.\begin{bmatrix} -X_1 \\ 0 \\ 0 \end{bmatrix}$$

(k) $\quad N(E) \quad = \begin{bmatrix} 0 & -Z_1 & 0 \\ Z_1 & 0 & -X_1 \\ 0 & X_1 & 0 \end{bmatrix}$

(l) $\quad M_e.E + \displaystyle\sum_e m.D.P_e \quad = \begin{bmatrix} M_e.X_1 + H_e.\cos\phi_h \\ V_e \\ M_e.Z_1 + H_e.\sin\phi_h \end{bmatrix}$

(m) $\displaystyle\sum_e m.\dot{D}.P_e = \dot{\phi}.\begin{bmatrix} -H_e.\sin\phi_h \\ 0 \\ H_e.\cos\phi_h \end{bmatrix}$

(n) $\displaystyle\sum_e m.\ddot{D}.P_e = \ddot{\phi}.\begin{bmatrix} -H_e.\sin\phi_h \\ 0 \\ H_e.\cos\phi_h \end{bmatrix} + \dot{\phi}^2.\begin{bmatrix} -H_e.\cos\phi_h \\ 0 \\ -H_e.\sin\phi_h \end{bmatrix}$

(o) $N(E).C^{-1}.\ddot{X}.C = \frac{1}{2}(\ddot{\eta}+j.\ddot{\psi}).e^{-j\theta}.\begin{bmatrix} 0 & 0 & Z_1 \\ -j.X_1 & -X_1 & -j.Z_1 \\ 0 & 0 & -X_1 \end{bmatrix} + \text{Conjugate}$

(p) $N(E).C^{-1}.\dot{X}.\dot{C}. = \frac{1}{2}(\dot{\eta}+j.\dot{\psi}).\dot{\theta}.e^{-j\theta}.\begin{bmatrix} 0 & 0 & 0 \\ -X_1 & j.X_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \text{Conjugate}$

(q) $N(E).C^{-1}.\ddot{C} = \ddot{\theta}.\begin{bmatrix} -Z_1 & 0 & 0 \\ 0 & -Z_1 & 0 \\ X_1 & 0 & 0 \end{bmatrix} + \dot{\theta}^2.\begin{bmatrix} 0 & Z_1 & 0 \\ -Z_1 & 0 & 0 \\ 0 & -X_1 & 0 \end{bmatrix}$

(r) $N(E).C^{-1}.\dot{X}.C = \frac{1}{2}(\dot{\eta}+j.\dot{\psi}).e^{-j\theta}.\begin{bmatrix} 0 & 0 & Z_1 \\ -j.X_1 & -X_1 & -j.Z_1 \\ 0 & 0 & -X_1 \end{bmatrix} + \text{Conjugate}$

(s) $N(E).C^{-1}.\dot{C} = \dot{\theta}.\begin{bmatrix} -Z_1 & 0 & 0 \\ 0 & -Z_1 & 0 \\ X_1 & 0 & 0 \end{bmatrix}$

(t) $\displaystyle\sum_e m.N(K).Q.R^{-1}.G = M_e.N(E).C^{-1}.(I-X).G$

(u) $\displaystyle\sum_{e} m.N(K).Q.R^{-1}.\ddot{P}' =$

$$N(E).(C^{-1}.\ddot{X}.C + 2C^{-1}.\dot{X}.\dot{C} + C^{-1}.\ddot{C}).\left(M_e.E + \sum_{e} m.D.P_e \right)$$

$$+ 2N(E).(C^{-1}.\dot{X}.C + C^{-1}.\dot{C}).\sum_{e} m.\dot{D}.P_e + N(E).\sum_{e} m.\ddot{D}.P_e$$

(v) $R^{-1}.\dot{R} = C^{-1}.\dot{X}.C + C^{-1}.\dot{C}$

(w) $\displaystyle\sum_{r} m.N(P).(R^{-1}.\dot{R}.P + \dot{P}) =$

$$\dot{\theta}.\begin{bmatrix} -L_r.\cos\gamma_r \\ -L_r.\sin\gamma_r \\ 2J_r \end{bmatrix} + \tfrac{1}{2}(\dot{\eta}+j.\dot{\psi}).e^{-j\theta}.\begin{bmatrix} J_r+Q_r+K_r.\cos\beta_r+j.K_r.\sin\beta_r \\ -j(J_r+Q_r)+j.K_r.\cos\beta_r-K_r.\sin\beta_r \\ j.L_r.\sin\gamma_r - L_r.\cos\gamma_r \end{bmatrix}$$

$$+ \text{Conjugate}$$

(x) $\displaystyle\sum_{r} m.N(P) = \begin{bmatrix} 0 & -V_r & H_r.\sin\alpha_r \\ V_r & 0 & -H_r.\cos\alpha_r \\ -H_r.\sin\alpha_r & H_r.\cos\alpha_r & 0 \end{bmatrix}$

(y) $R^{-1}.G = C^{-1}.(I-X).G$

APPENDIX II

KEY RESULTS IN THE EVALUATION OF EQUATIONS (72) AND (73)

(a) $\quad \Delta(R^{-1}.\dot{R}) \quad = \quad D^{-1}.M^{-1}.C^{-1}.(\dot{C}.M.D + C.M.\dot{D}) - D^{-1}.C^{-1}.(\dot{C}.D + C.\dot{D})$

$\qquad\qquad\quad \simeq \quad D^{-1}.\{C^{-1}.\dot{C}.(M-I) - (M-I).C^{-1}.\dot{C}\}.D$

$\qquad\qquad\quad \simeq \quad D\eta.\dot{\theta}. \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(b) $\quad \displaystyle\sum_{e} m.N(P).\Delta(R^{-1}.\dot{R}).P \quad \simeq \quad D\eta.\dot{\theta}. \begin{bmatrix} -L_e.\cos\gamma_e \\ 2J_e \\ -L_e.\sin\gamma_e \end{bmatrix}$

(c) $\quad D.\Delta(R^{-1}) \quad = \quad D.D^{-1}.M^{-1}.C^{-1} - D.D^{-1}.C^{-1}$

$\qquad\qquad\quad \simeq \quad - (M-I).C^{-1}$

(d) $\quad D.\Delta(R^{-1}).G \quad \simeq \quad D\eta.g. \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(e) $\quad D.\Delta(R^{-1}).\ddot{L} \quad \simeq \quad - (M-I).C^{-1}.\ddot{C}.E$

$\qquad\qquad\quad \simeq \quad \ddot{\theta}.X_1. \begin{bmatrix} D\theta \\ 0 \\ -D\eta \end{bmatrix} + \dot{\theta}^2.X_1. \begin{bmatrix} 0 \\ D\theta \\ 0 \end{bmatrix}$

(f) $\quad \Delta(Q) \quad = \quad (M-I).D$

$$= D\eta . \begin{bmatrix} 0 & 0 & 0 \\ -c & 0 & -d \\ 0 & 1 & 0 \end{bmatrix} + D\theta . \begin{bmatrix} 0 & -1 & 0 \\ d & 0 & -c \\ 0 & 0 & 0 \end{bmatrix}$$

(g) $\quad Q \quad = \quad D \quad = \begin{bmatrix} d & 0 & -c \\ 0 & 1 & 0 \\ c & 0 & d \end{bmatrix}$

(h) $\quad \Delta(Q.R^{-1}) \quad = \quad M.D.D^{-1}.M^{-1}.C^{-1} - D.D^{-1}.C^{-1} \quad = \quad 0$

(i) $\quad \Delta(Q.R^{-1}.\ddot{P}') \quad = \quad \Delta(C^{-1}.\dfrac{d^2}{dt^2}\{C.E + C.M.D.P_e\})$

$$= \quad C^{-1}.\dfrac{d^2}{dt^2}\{C.(M-I).D.P_e\}$$

$$= \quad C^{-1}.\dfrac{d}{dt}\{(\dot{C} + C.\dfrac{d}{dt})((M-I).D.P_e)\}$$

$$= \quad (C^{-1}.\dot{C} + \dfrac{d}{dt})\{(C^{-1}.\dot{C} + \dfrac{d}{dt})\{(M-I).D.P_e\}\}$$

(j) $\quad \displaystyle\sum_e m.(M-I).D.P_e$

$$= \quad D\eta . \begin{bmatrix} 0 \\ -H_e.\sin\phi_h \\ V_e \end{bmatrix} + D\theta . \begin{bmatrix} -V_e \\ H_e.\cos\phi_h \\ 0 \end{bmatrix}$$

(k) $\quad N(K) \qquad = \quad N(E) = \begin{bmatrix} 0 & -Z_1 & 0 \\ Z_1 & 0 & -X_1 \\ 0 & X_1 & 0 \end{bmatrix}$

APPENDIX III

KEY RESULTS IN EVALUATION OF SECTION 14

(a) $\quad C. \displaystyle\sum_{r,e} m.P_r \;=\; [H.\cos\theta \quad H.\sin\theta \quad V]^T$

(b) $\quad (\ddot{X}-\ddot{Y}).E_2 \;=\; [0 \quad -\ddot{\eta}.Z_2 \quad (\ddot{\eta}.Y_2-\ddot{\psi}.X_2)]^T$

(c) $\quad \Delta\Gamma_s \;=\; \ddot{\eta}.\begin{bmatrix} -V.Z_2 - Y_2.H.\sin\theta_h \\[1em] Y_2.H.\cos\theta_h \\[1em] Z_2.H.\cos\theta_h \end{bmatrix} + \ddot{\psi}.\begin{bmatrix} X_2.H.\sin\theta_h \\[1em] -X_2.H.\cos\theta_h \\[1em] 0 \end{bmatrix}$

(d) $\quad [0 \; 1 \; 0].D^{-1}.C^{-1} \;=\; [-\sin\theta \quad \cos\theta \quad 0]$

(e) $\quad C.E \;=\; [X_1.\cos\theta \quad X_1.\sin\theta \quad Z_1]^T$

(f) $\quad [0 \; 1 \; 0].D^{-1}.C^{-1}.N(C.E) \;=\; [Z_1.\cos\theta \quad Z_1.\sin\theta \quad -X_1]$

(g) $\quad \Delta T_{elev} \;=\; \ddot{\eta}.\{(V-M_e.Z_1).Z_2.\sin\theta - Y_2.(M_e.X_1-H.\cos\alpha)\}$
$\qquad\qquad\qquad + \ddot{\psi}.X_2.(M_e.X_1-H.\cos\alpha)$

(h) $\quad \displaystyle\sum_{r,e} m.P_s \;=\; \begin{bmatrix} H.\cos\theta_h \\[1em] H.\sin\theta_h \\[1em] V \end{bmatrix}$

(i) $\quad \displaystyle\sum_{r,e} m.\dot{P}_s \;=\; \begin{bmatrix} -\dot\theta.H.\sin\theta_h - \dot\phi.H_e.\cos\theta.\sin\phi_h \\[1em] \dot\theta.H.\cos\theta_h - \dot\phi.H_e.\sin\theta.\sin\phi_h \\[1em] 0 \end{bmatrix}$

(j) $\displaystyle\sum_{r,e} m.\ddot{P}_s = \begin{bmatrix} -\ddot{\theta}.H.\sin\theta_h - \dot{\theta}^2.H.\cos\theta_h + 2\dot{\theta}.\dot{\phi}.H_e.\sin\theta.\sin\phi_h \\[6pt] - \ddot{\phi}.H_e.\cos\theta.\sin\phi_h - \dot{\phi}^2.H_e.\cos\theta.\cos\phi_h \\[24pt] \ddot{\theta}.H.\cos\theta_h - \dot{\theta}^2.H.\sin\theta_h - 2\dot{\theta}.\dot{\phi}.H_e.\cos\theta.\sin\phi_h \\[6pt] - \ddot{\phi}.H_e.\sin\theta.\sin\phi_h - \dot{\phi}^2.H_e.\sin\theta.\cos\phi_h \\[24pt] 0 \end{bmatrix}$

(k) $\quad K_b = \begin{bmatrix} X_2 \\[6pt] Y_2 - \eta.Z_2 \\[6pt] Z_2 + \eta.Y_2 - \psi.X_2 \end{bmatrix}$

(l) $\quad [1\ 0\ 0].N(K_b) \qquad\qquad\qquad = \quad [0 \quad -(Z_2+\eta.Y_2-\psi.X_2) \quad (Y_2-\eta.Z_2)]$

(m) $\quad [1\ 0\ 0].N(E_2).X \qquad\qquad\quad = \quad [-\psi.Y_2 \quad -(Z_2-\psi.X_2) \quad Y_2]$

(n) $\quad [1\ 0\ 0].(I+X).N\{(I-X).K_b\} \quad = \quad [-\psi.Y_2 \quad -(Z_2-\psi.X_2) \quad Y_2]$

(o) $\quad [0\ 1\ 0].N(K_g) \qquad\qquad\qquad = \quad [-\psi.X_2 \quad 0 \quad -X_2]$

(p) $\quad [0\ 1\ 0].N(B^{-1}.K_g).X \qquad\quad = \quad [\psi.X_2 \quad -\eta.X_2 \quad -X_2]$

(q) $\quad [0\ 1\ 0].(I+X).N\{(I-X).K_g\} \quad = \quad [0 \quad -\eta.X_2 \quad -X_2]$
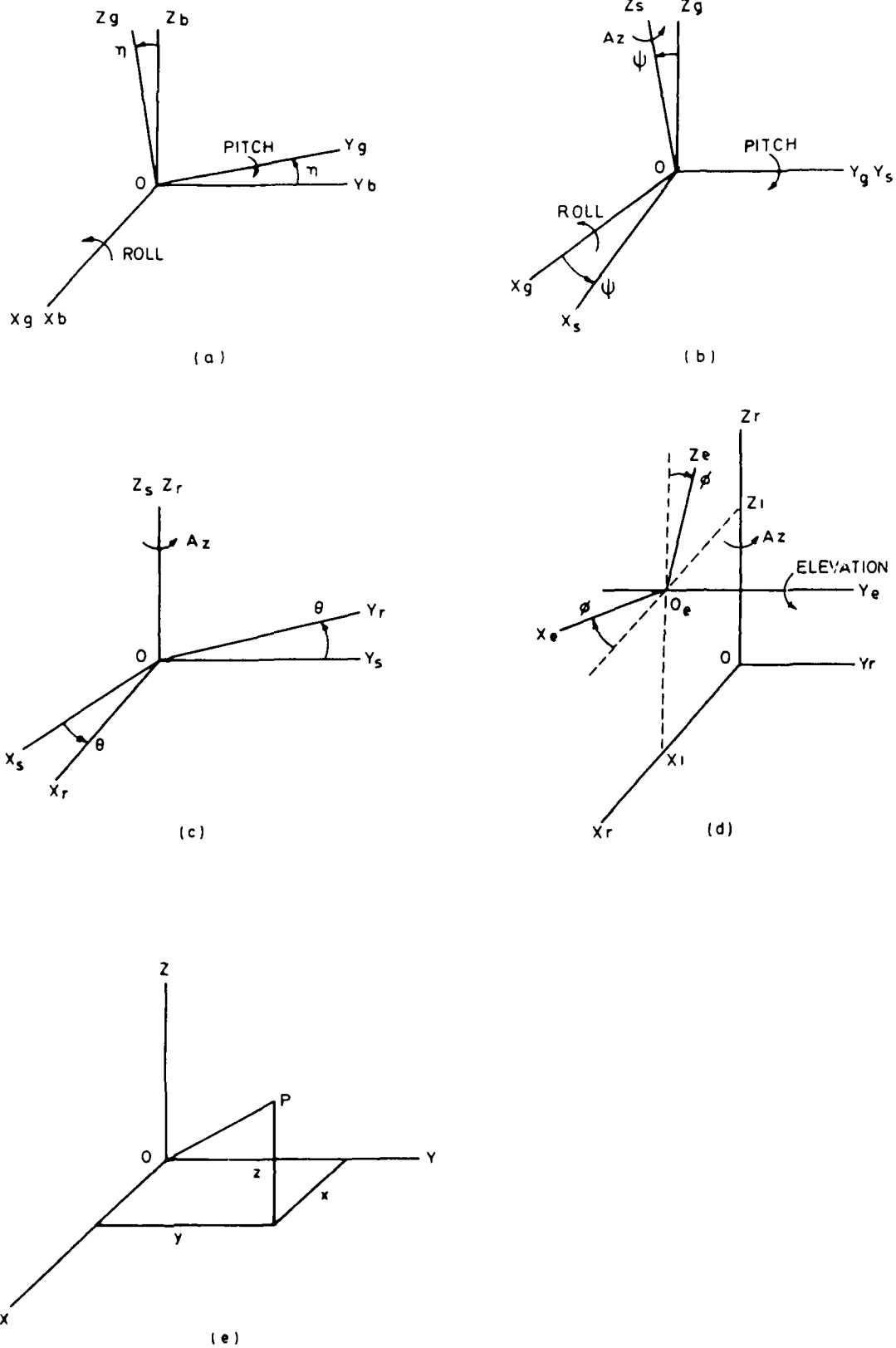
Figure 1.   Relationship of axes
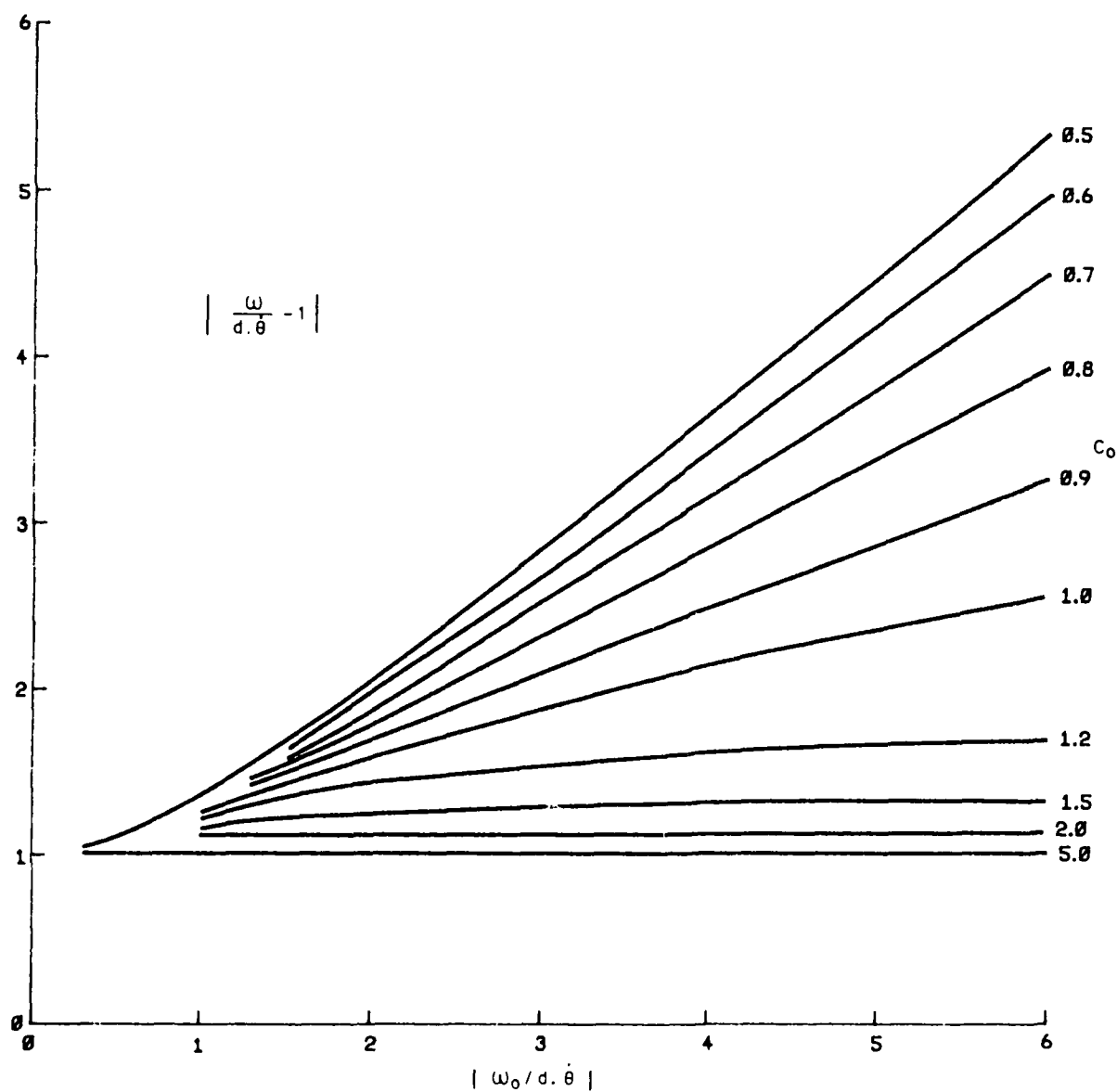
Figure 2.   Properties of transient solution of stabilisation system
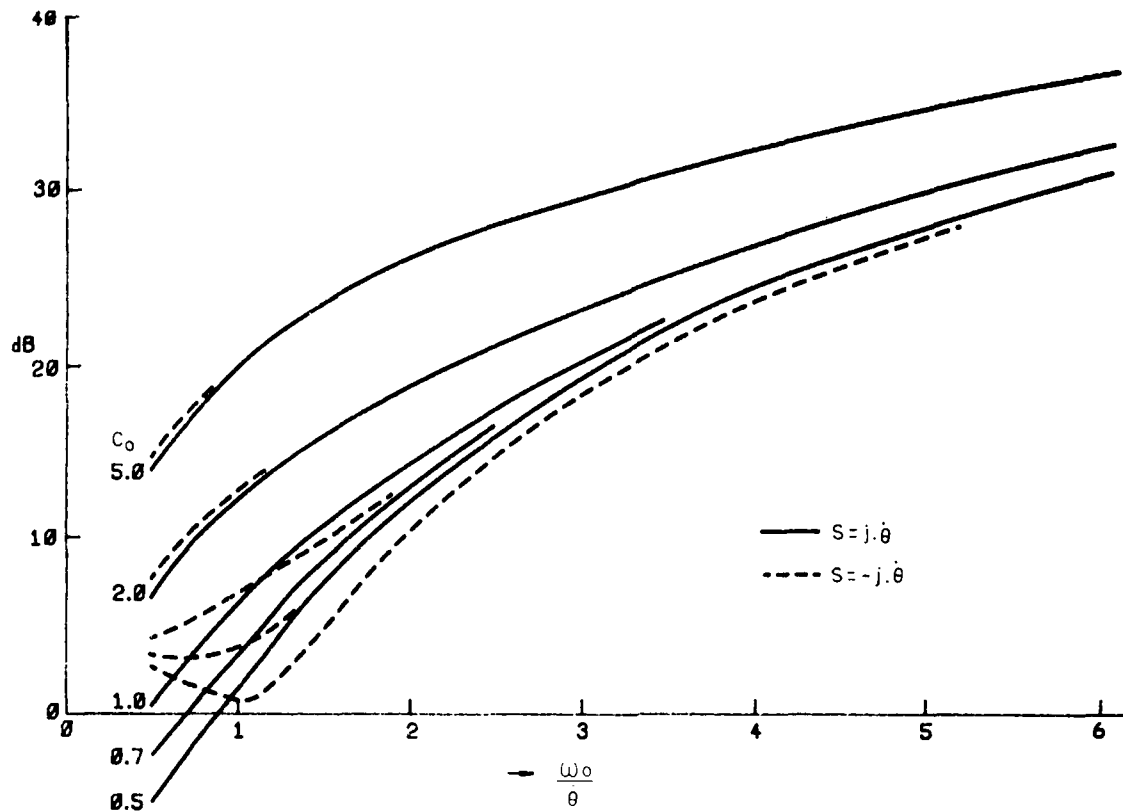
(a) $d = \dfrac{J}{J+Q} = 0.25$
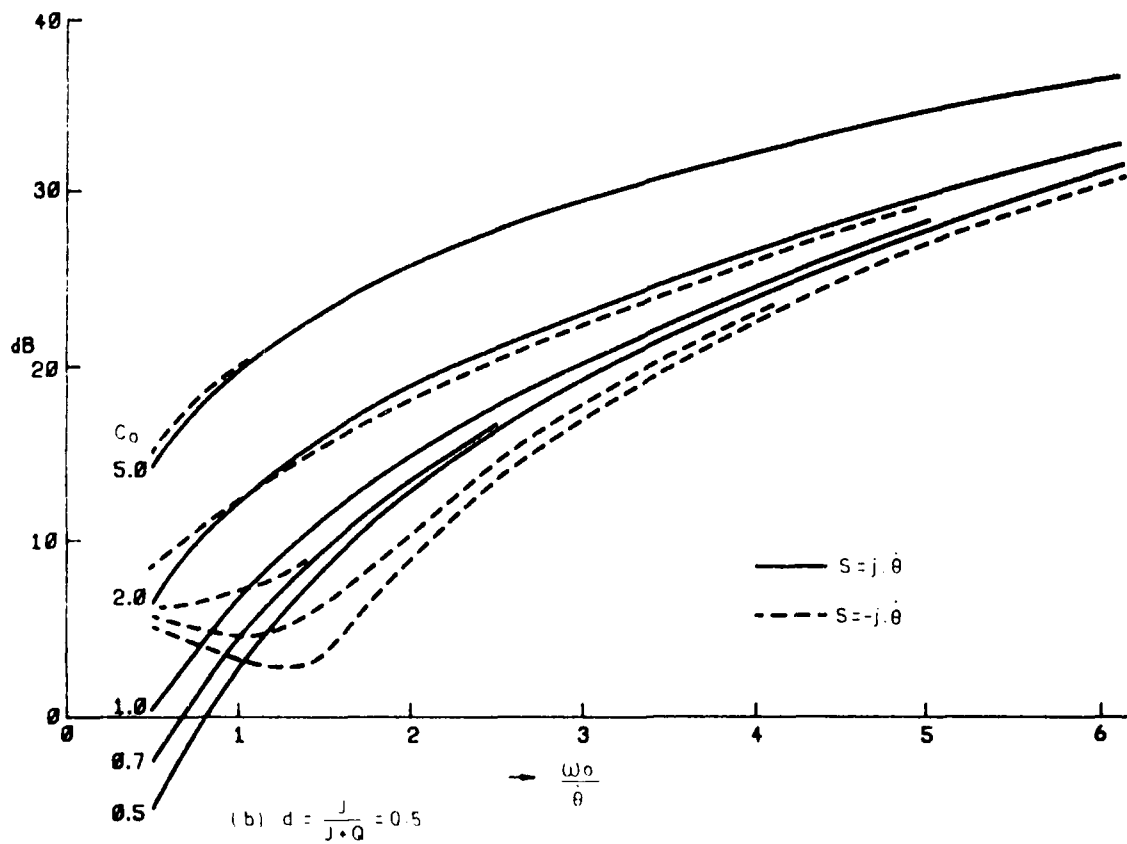
(b) $d = \dfrac{J}{J+Q} = 0.5$

Figure 3.   Magnitude of $G(s)/\dot{\theta}^2$

The official documents produced by the Laboratories of the Defence Research Centre Salisbury are issued in one of five categories: Reports, Technical Reports, Technical Memoranda, Manuals and Specifications. The purpose of the latter two categories is self-evident, with the other three categories being used for the following purposes:

Reports   :   documents prepared for managerial purposes.

Technical   :   records of scientific and technical work of a permanent value intended for other
Reports   scientists and technologists working in the field.

Technical   :   intended primarily for disseminating information within the DSTO. They are
Memoranda   usually tentative in nature and reflect the personal views of the author.

Security classification of this page: [UNCLASSIFIED]

13 | ANNOUNCEMENT LIMITATIONS (of the information on these pages):

No limitations.

14 | DESCRIPTORS:

a. EJC Thesaurus
Terms

Stabilised platforms
Mounts
Balance
Torque
Weapon systems

b. Non-Thesaurus
Terms

15 | COSATI CODES:

14020

16 | SUMMARY OR ABSTRACT:
(if this is security classified, the announcement of this report will be similarly classified)

The four-axis, elevation over azimuth over pitch over roll, director mount
is analysed to determine the torques generated by mass unbalances about the
motor driven axes. The effect of these on the motion of the stabilised
platform, and the effect of this motion, as well as that of mechanical
tolerances, on the accuracy of direction is examined. The analysis is
restricted to the case of the fully erect system, firmly connected to a
stationary, level surface, but the method of approach could be extended to
more complex situations. The results provide the basis for the design of
small, lightweight, high performance directors.

DOCUMENT CONTROL DATA SHEET

1	DOCUMENT NUMBERS	2	SECURITY CLASSIFICATION
AR Number: AR-003-772		a. Complete Document: Unclassified	
Series Number: WSRL-0355-TR		b. Title in Isolation: Unclassified	
Other Numbers:		c. Summary in Isolation: Unclassified	

3 TITLE

AN ANALYSIS OF THE EFFECT OF MASS UNBALANCES AND ASSEMBLY TOLERANCES ON THE PERFORMANCE OF THE FOUR-AXIS STABILISED DIRECTOR MOUNT

4 PERSONAL AUTHOR(S):	5 DOCUMENT DATE:
J. Hayward	May 1984
	6 6.1 TOTAL NUMBER OF PAGES 68
	6.2 NUMBER OF REFERENCES:

7 7.1 CORPORATE AUTHOR(S):	8 REFERENCE NUMBERS
Weapons Systems Research Laboratory	a. Task: DST 78/057
	b. Sponsoring Agency:
7.2 DOCUMENT SERIES AND NUMBER Weapons Systems Research Laboratory 0355-TR	9 COST CODE: 358-AA-320

10 IMPRINT (Publishing organisation)	11 COMPUTER PROGRAM(S) (Title(s) and language(s))
Defence Research Centre Salisbury	

12 RELEASE LIMITATIONS (of the document):

Approved for Public Release

DISTRIBUTION

Copy No.

DEPARTMENT OF DEFENCE

Defence Science and Technology Organisation

Chief Defence Scientist

Deputy Chief Defence Scientist

Controller, External Relations, Projects and Analytical Studies   } 1

Superintendent, Science Programs and Administration

Defence Science Representative, London   Title page

Counsellor, Defence Science, Washington   Title page

Weapons Systems Research Laboratory

Director, Weapons Systems Research Laboratory   2

Superintendent, Weapon Systems Division   3

Head of Composite, Marine Studies   4

Principal Officer, Tracking and Command Systems Group 5

Principal Officer, Systems Integration Group   6

Principal Officer, Systems Modelling Group   7

Principal Officer, Terminal Guidance Group   8

J.Hayward, Tracking and Command Systems Group   9

A.R.Tonissen, Sea Experiments Group   10

Advanced Engineering Laboratory

Director, Advanced Engineering Laboratory   11

Principal Engineer, Mechanical Engineering Design Branch   12

W.R.Crook, Mechanisms and Instrumentation Group   13

Aeronautical Research Laboratories

Director/Library, Aeronautical Research Laboratories 14

Superintendent, Systems Division   15

Materials Research Laboratories

Director/Library, Materials Research Laboratories   16

Navy Office
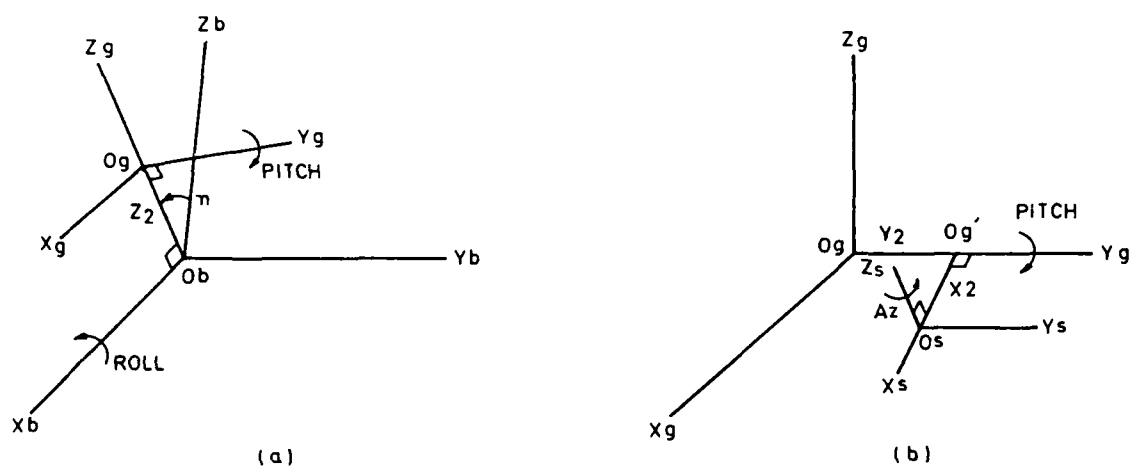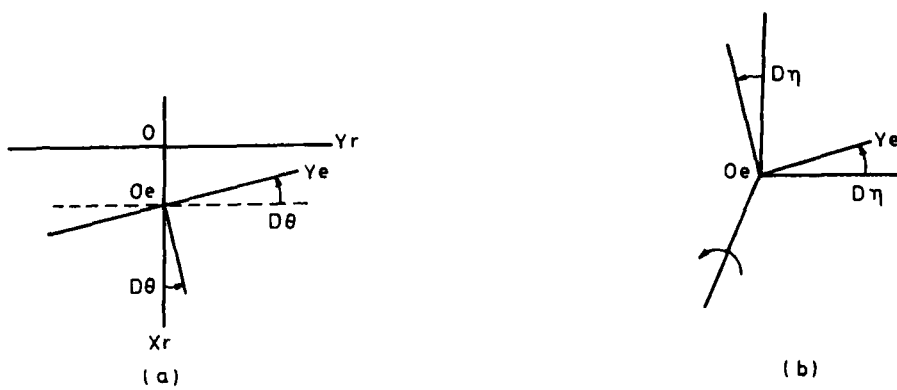
Figure 7. Non-intersecting roll, pitch and azimuth axes
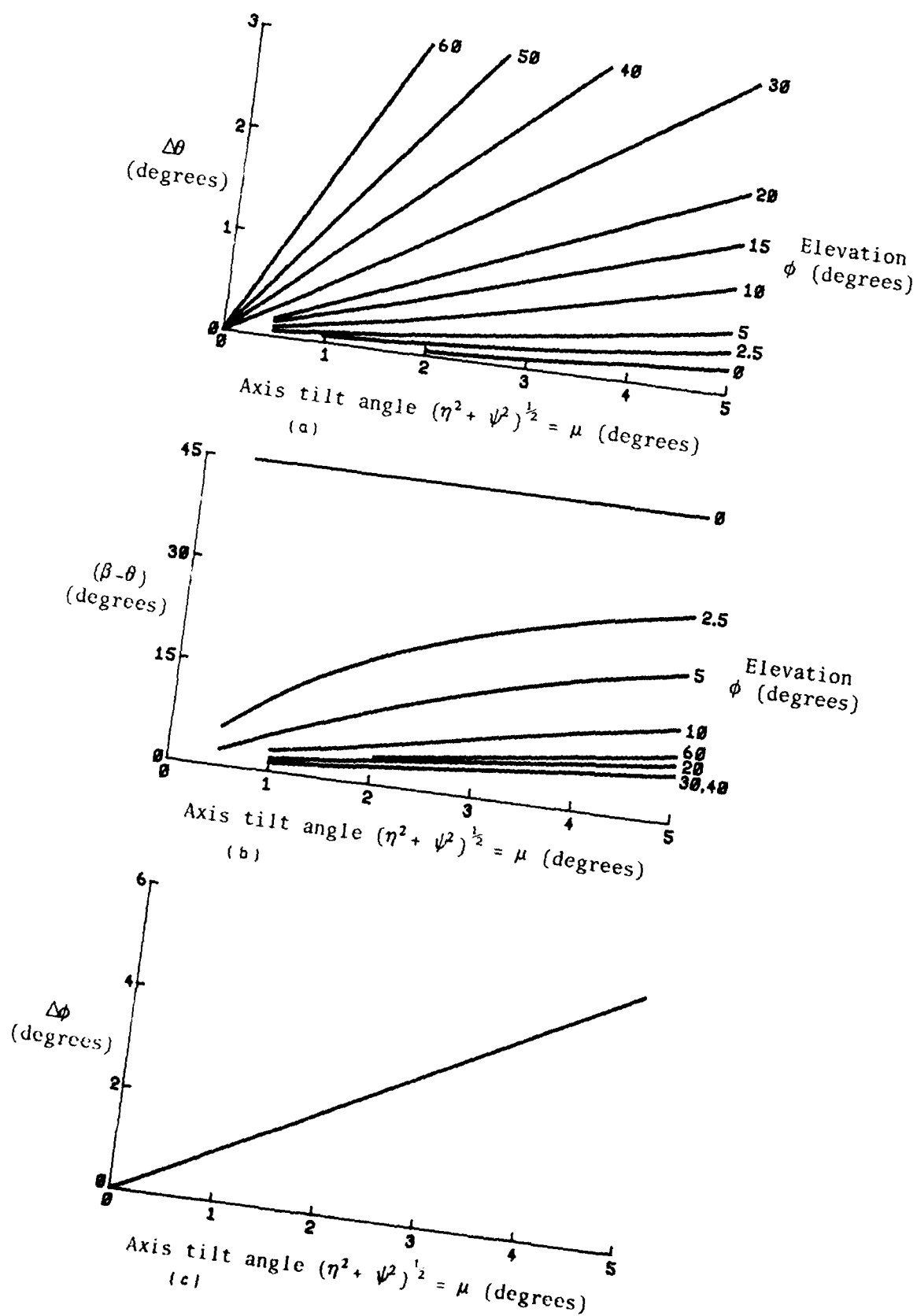
Figure 6. Tilted elevation axis, O_eY_e

Figure 5. Maximum boresight errors versus tilt angle
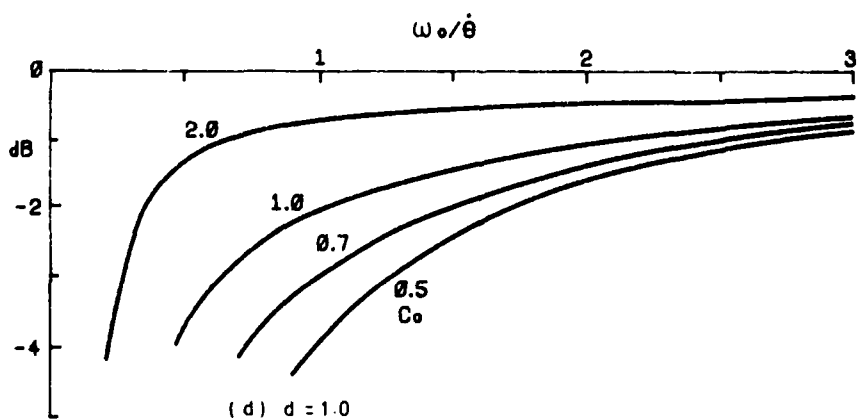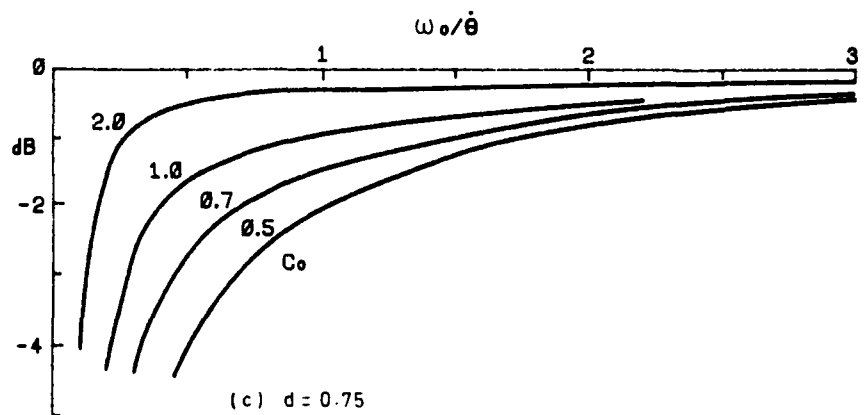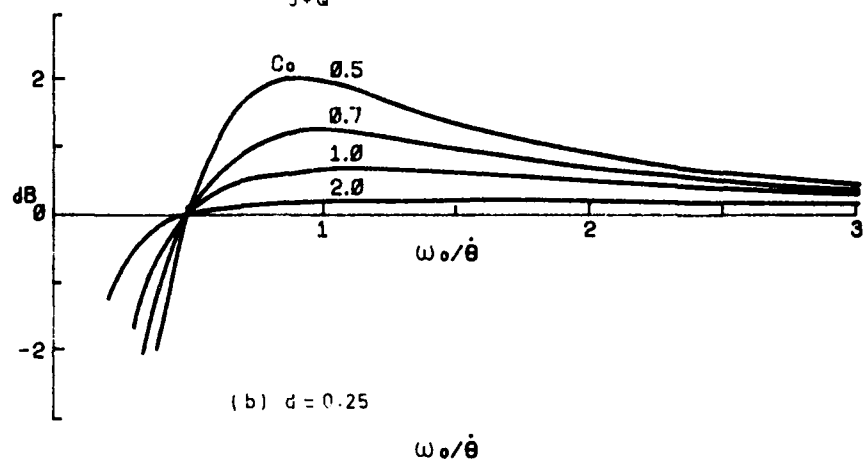
(a) $\dfrac{J}{J+Q} = d = 0.1$

(b) $d = 0.25$

(c) $d = 0.75$

(d) $d = 1.0$
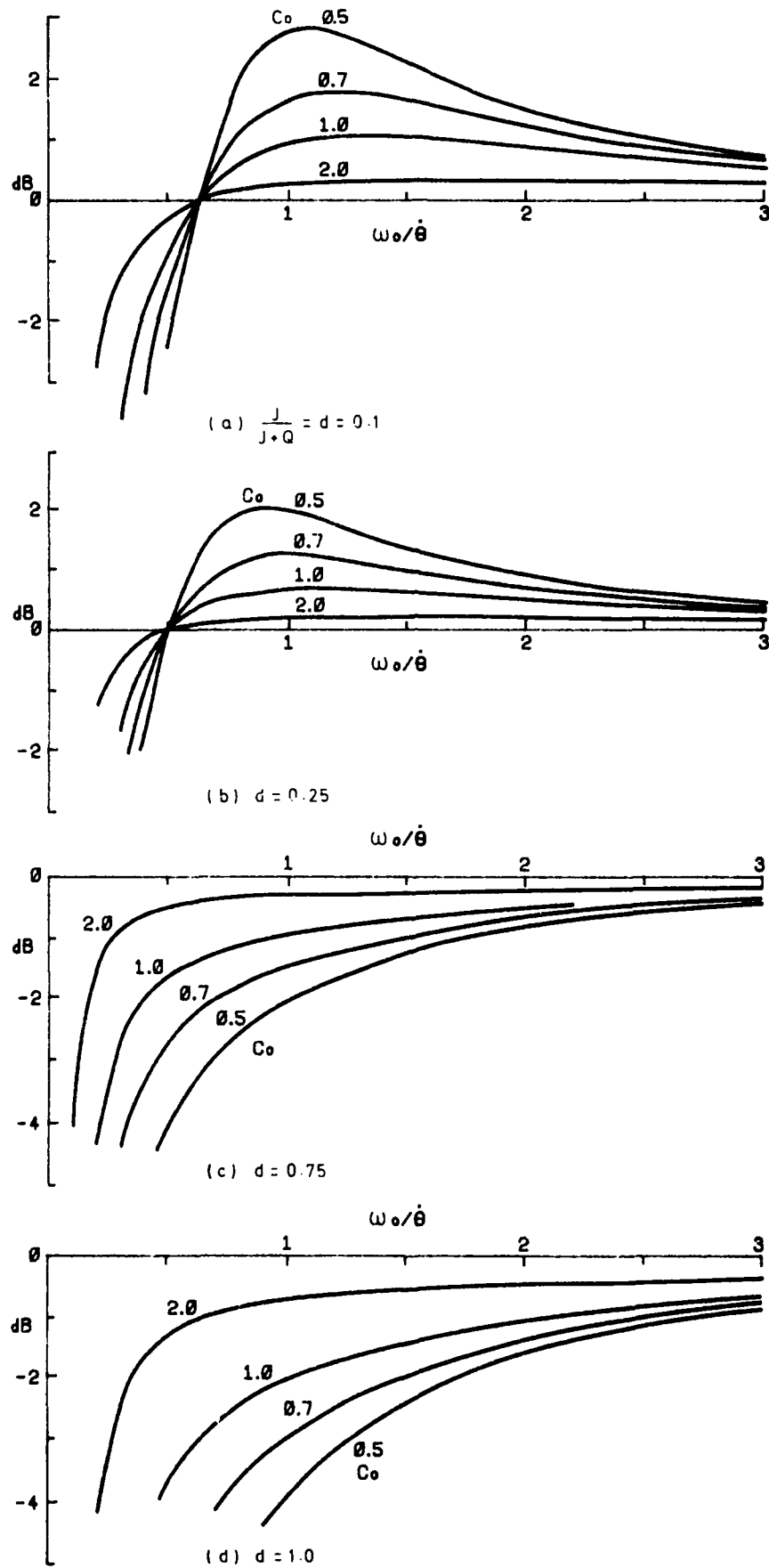
Figure 4.   Ratio of motor to forcing torque

(d) $d = \dfrac{J}{J+Q} = 0.25$ $\longrightarrow \dfrac{\omega_o}{\dot{\theta}}$
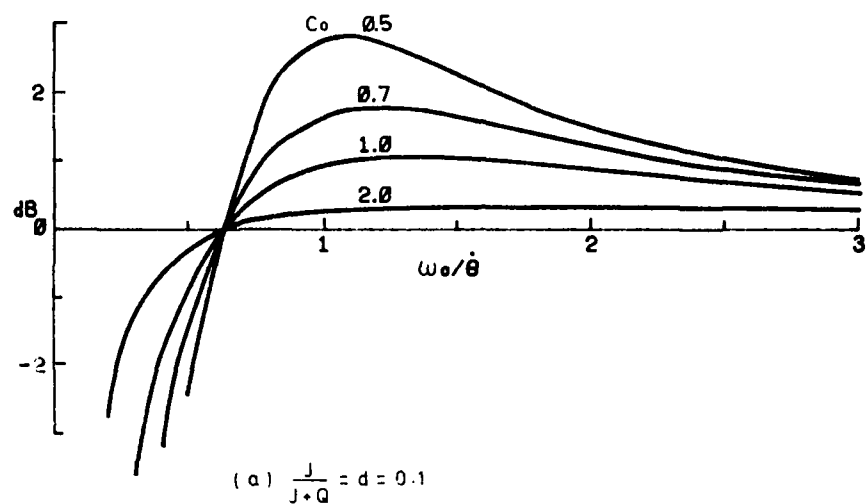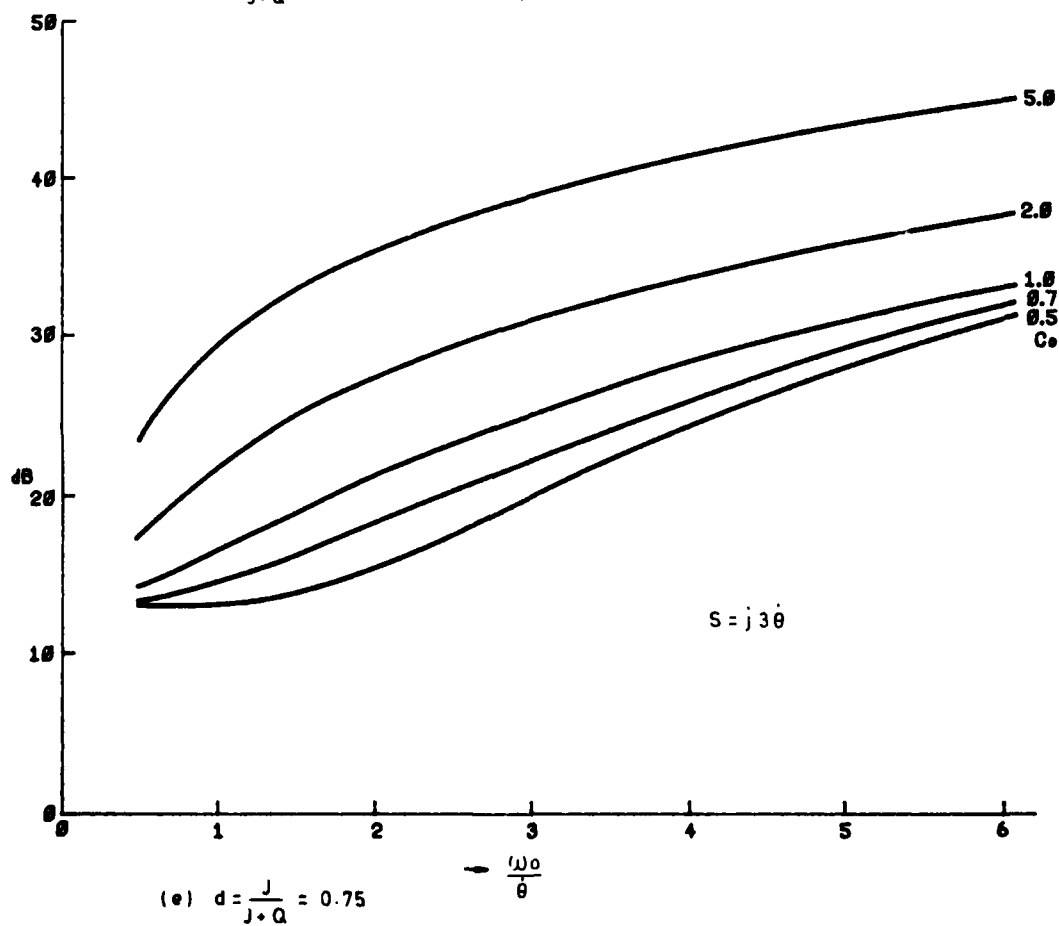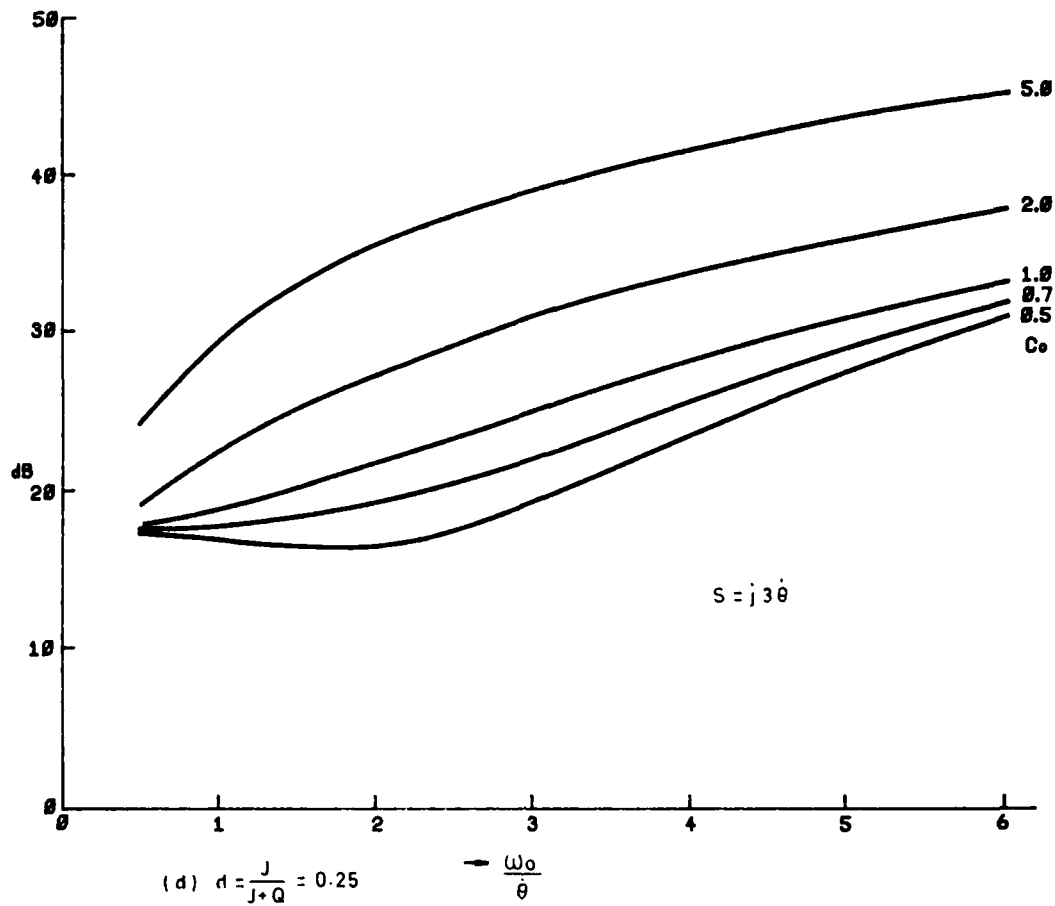
(e) $d = \dfrac{J}{J+Q} = 0.75$ $\longrightarrow \dfrac{\omega_o}{\dot{\theta}}$

Figure 3(Contd.)

Figure 4. Ratio of motor to forcing torque

(d) $d = \dfrac{J}{J+Q} = 0.25$

$S = j\,3\dot{\theta}$

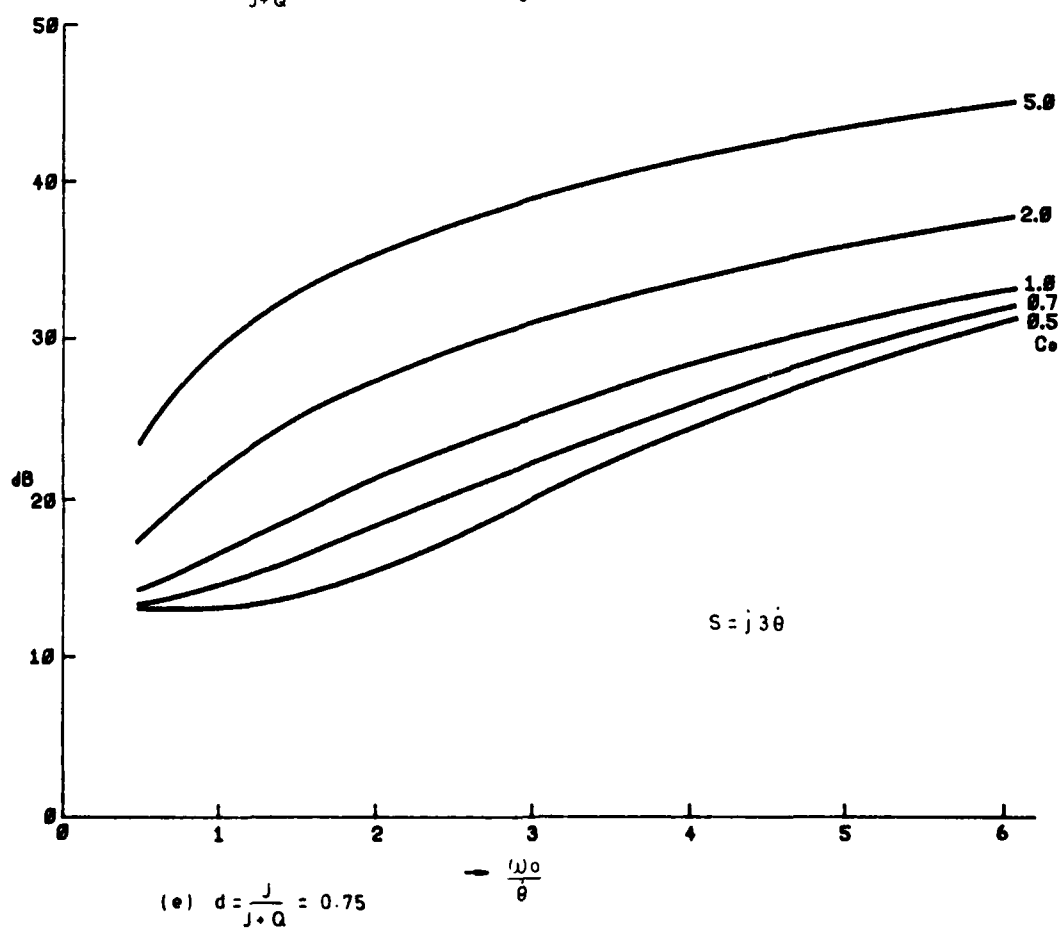(e) $d = \dfrac{J}{J+Q} = 0.75$
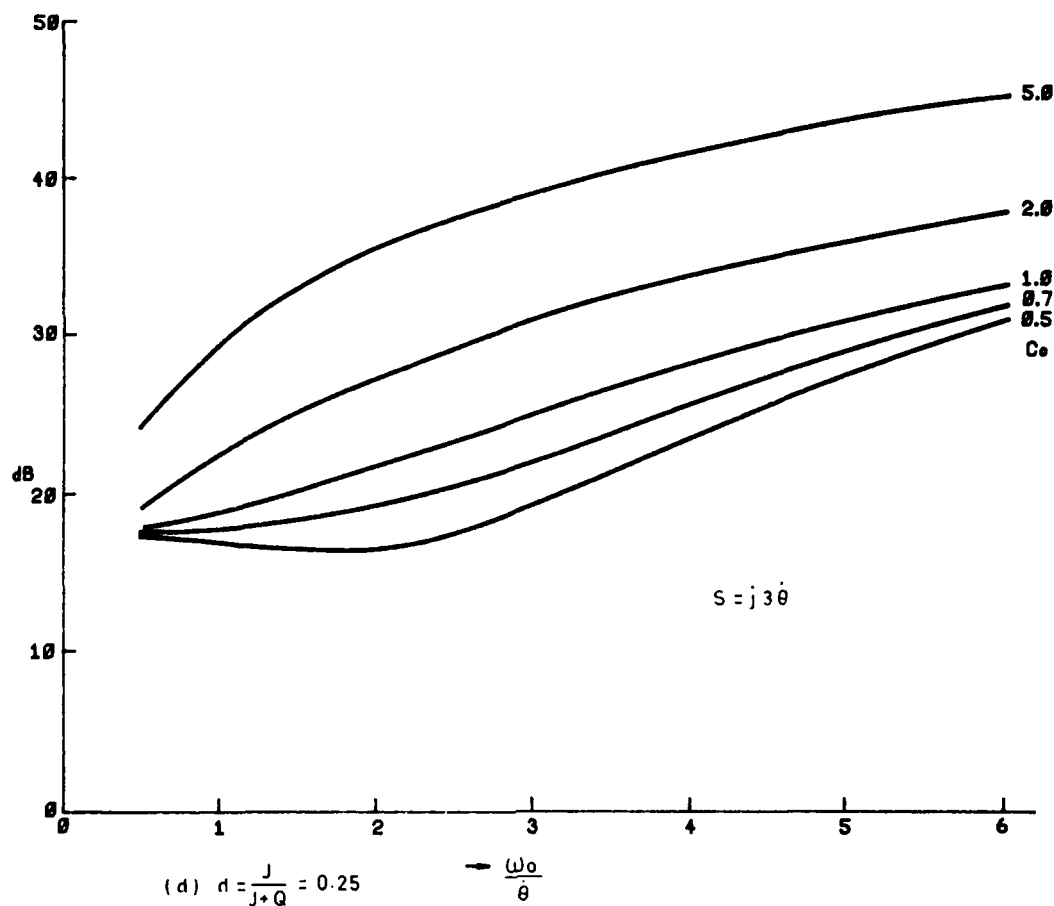
$S = j\,3\dot{\theta}$

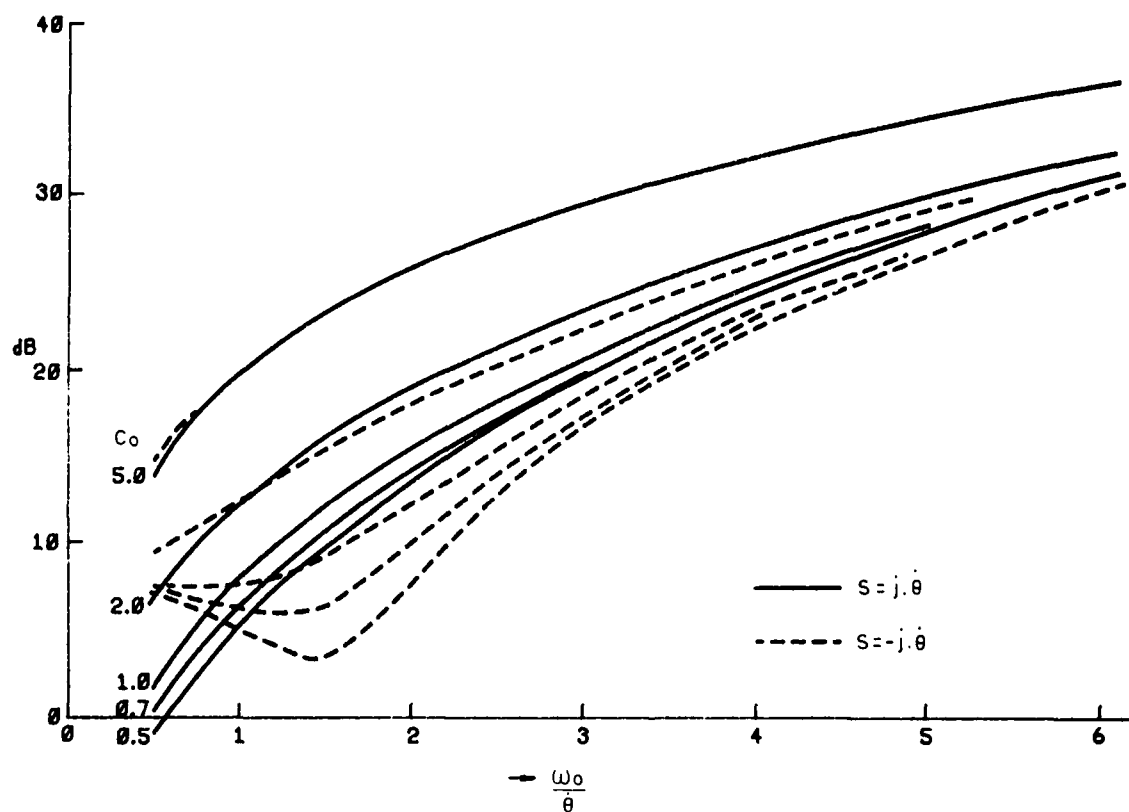Figure 3(Contd.)

(c) $d = \dfrac{J}{J+Q} = 0.75$

Figure 3(Contd.).

END

FILMED

4-85

DTIC